A COMPARISON OF BUFFER STRIP AND NON-BUFFER STRIP JOINT DESIGNS

James Michael Gill

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THESIS

A COMPARISON OF BUFFER STRIP AND NON-BUFFER STRIP JOINT DESIGNS

by

James Michael Gill

June 1976

Thesis Advisor: M.H. Bank II

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LIST OF SYMBOLS

AD = overall buffer strip joint width (in.)

A_n = constant of proportionality between the bolt load P and the portion of P reacted each of the primary strips of a buffer strip joint

B = excess bearing capacity

D = bolt hole diameter (in.)

E_x = tensile modulus (lbf./in.^{2})

E_{x_1} = tensile modulus of the primary strips in a buffer strip joint (lbf./in.^{2})

E_{x_2} = tensile modulus of the buffer strip material in a buffer strip joint (lbf./in.^{2})

F_{BP} = bypass force (lbf.)

F_S = shear load passing P_1 from a buffer strip to a primary strip (lbf.)

F_r = reaction force (lbf.)

$f(\frac{a}{r})^{i}_{\alpha}$ = effective isotropic stress concentration factor at location i for load condition α

f_s = shear stress between the buffer and primary strips of a buffer strip joint (lbf./in.^{2})

i = indicator of exact position on the hole

L = tensile load (lbf.)

ℓD = side length (in.)

M = number of rows of bolts in a buffer strip joint

m = reaction moment (in.-lbf.)

N = number of rows of bolts in a non-buffer strip joint

N_x = tensile load (lbf./in.)

N_{xy} = shear load (lbf./in.)

P = tensile bolt load (lbf.)

P_s = shear bolt load (lbf.)

P_1 = portion of P reacted in each of the primary strips of a buffer strip joint (lbf.)

P_2 = portion of P reacted in the buffer strip material of a buffer strip joint (lbf.)

R = resultant bolt load (lbf.)

S_1 = representative applied stress (lbf./in.^{2})

S_2 = representative failure stress (lbf./in.^{2})

t = plate thickness (in.)

t^* = effective plate thickness (in.)

$t_{\pm 45}$ = total thickness of $\pm 45^{\circ}$ laminae (in.)

$W_B D$ = width of buffer strip material (in.)

$W_1 D$ = half-width of primary laminate material in a buffer strip joint (in.)

Z = percentage of zero degree plies in the primary strips of a buffer strip joint

α = subscript denoting applied load conditions as follows:

= bx bearing in x direction
= by bearing in y direction
= tx tension in x direction
= ty tension in y direction
= xy shear

ϵ = strain (in./in.)

λ_{α}^{i} = finite width correction factor at location i for

applied load condition α

η = indicator of position in a joint

σ_{BR} = resultant bolt bearing stress (lbf./in.^{2})

σ^i = net tangential stress at location i

σ_α = applied stress for load condition α

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I. INTRODUCTION

A. BACKGROUND

Aerospace structural design requirements have often been characterized by a demand for high strength and low weight. In many cases, the design possibilities have been limited by the available manufacturing technology and the acceptable manufacturing costs. The most successful designs were usually those which met all these constraints most efficiently.

The demand for high structural efficiency led to the development of advanced laminated composites which have higher strength-to-weight ratios and better fatigue properties than conventional structural materials (Ref. 1). As discussed in Ref. 2, it has also been possible to build laminated composite materials with higher modulus values than those characteristic of conventional materials. Accurate methods for tailoring the properties of such materials have evolved and such tailoring has become accepted design practice (Refs. 3,4,5). Waddoups, in Ref. 6, explained that significant gains in structural efficiency have been demonstrated by doing no more than substituting such a tailored composite for a conventional material keeping the geometry and mating interfaces the same.

These gains were realized because the composite materials used were less dense than the conventional

structural materials which they replaced. These material substitutions did not exploit the high strength and high modulus values achievable with advanced laminated composites.

B. OBJECTIVES

This study was intended to demonstrate the structural efficiencies which could be achieved by designs which took into account some of the high strength and high modulus properties of advanced composites. It was decided to demonstrate these efficiencies by analyzing the behavior of a plate of laminated material in the region of a bolted joint. This situation corresponded to that of a wing skin attached by a bolted fitting to the fuselage of an aircraft.

It was assumed that the wing skin would be attached to the aircraft fuselage through an aluminum alloy fitting. A maximum interbolt strain level, ϵ , of 3000 micro-inches per inch and an inter-bolt spacing of four hole diameters were taken as representative of such fittings.

Design conditions representative of an advanced fighter type aircraft were chosen. The joint was to carry a tensile load, N_x, of 20,000 lbf. per inch of chord. It was assumed that the joint fittings would be covered by an aerodynamic fairing, and a maximum joint length of ten inches was allowed.

To keep the joint manufacture as simple as possible, it was decided that the skin thickness would vary linearly in the joint and that for any given joint design, all holes

would be of the same diameter. To simplify the analysis it was decided to limit the candidate materials to balanced design laminates composed of zero and ±45 degree plies of uniform thickness and material composition. No attempt was made to judge the effect of these restrictions on design efficiency.

The interface requirements determined by the aluminum fitting, and the geometric requirements to satisfy manufacturing simplicity, limited the possible joint designs and prevented full utilization of the high strength and high modulus properties available in the selected materials. It was felt, however, that even under these restrictions it would still be possible to demonstrate significant structural efficiencies by properly tailoring the laminates used in the wing skin.

C. RELIABILITY CONSIDERATIONS

Aerospace structural designs have had to meet difficult requirements for reliability. These designs have had to be sufficiently strong to carry the required loads and light enough to work in the aerospace environment. In addition to these requirements, critical components of aerospace structures have been expected to demonstrate that they are "fail-safe" and, in military applications at least, to some degree battle damage tolerant.

Early composite materials were judged inadequate for aerospace applications because they could not meet these reliability conditions. At first, quantity production of

advanced laminates was impossible because the required quality control technology did not exist. Wide batch-to-batch variation of the properties of these early materials justified only low confidence levels in their structural reliability. Although the early advanced composites were considered unreliable, materials research continued, driven by the anticipated structural efficiencies such materials could make possible. In 1973, Kaminski reported that NARMCO 5208/T300 graphite-epoxy laminates could meet the anticipated requirements for high strength, high modulus, and low weight and could be manufactured reliably and delivered with minimal batch-to-batch variation of material properties (Ref. 2). This was chosen as the structural material to be used in this study.

The requirement for battle damage tolerance and fail-safe design is met by a variety of methods in designs with conventional materials. These methods include built in high excess load bearing capacity, alternate load paths, and various crack stoppage stress relief devices. All of these techniques can be applied to design with composite materials. In addition to these techniques there is a "buffer strip" material fabrication technique applicable to laminated composites which provides an integral crack stoppage capability. Kaminski and Eisenmann explained this technique in Ref. 7.

In buffer strip design, integral "buffer strips" of low modulus, high fracture toughness material are included

in the laminate. These strips are spaced so that cracks originating in the high modulus load bearing primary strips are arrested when they run into the buffer strips. Using this technique, structures can be built in which cracks would be arrested before entire structural components failed. It appears that this could be an effective and efficient way to increase battle damage tolerance and the capability of a structural component to function after crack initiation.

D. RANGE OF THE STUDY

NARMCO 5208/T300 $[0/\pm45]$ buffer strip and non-buffer strip joints were designed for widely varying laminate compositions, numbers and sizes of bolts. To simplify joint fabrication, the joint thickness was varied linearly between the inboard and outboard thicknesses, and all bolt holes for any given joint design were of the same diameter. To interface with the aluminum alloy fittings, the interbolt strain level was held to a maximum of 3000 micro-inches per inch, and the interbolt spacing was set at four hole diameters. The designs were compared to determine the effects of variation of hole size, number of holes, and laminate composition upon joint weight and excess bearing capacity.

Joint weight was considered a measure of the joint structural efficiency. Joint excess bearing capacity was considered a measure of allowable fabrication error. Since drilling holes in fibrous laminated composites has been

found to be both difficult and expensive, it was felt that the number of holes in any joint would be a measure of the relative joint fabrication cost (Ref. 5).

II. ANALYSIS OF THE NON-BUFFER STRIP JOINT

A. SIZING THE NON-BUFFER STRIP JOINT

Figure 1 is a schematic drawing showing the wing skin configuration used in the non-buffer strip joint design. All bolt holes are of the same diameter. For this analysis, wing taper is disregarded and the bolt hole centers are assumed placed in parallel rows and columns four hole diameters apart.

In the following theoretical development, it is assumed that the applied tensile load, N_x, and the applied shear load, N_{xy}, are constant across the outboard edge of the joint. Thus it is possible to size the joint considering only one column of bolts. It is further assumed that fittings were designed so that each bolt transfers the same portion, P, of the applied tensile load and that each bolt in the inboard row transfers, in addition to P, the same portion P_s of the shear load. In actual practice it is doubtful that this idealization could be achieved. However, it is a standard design assumption used in industry today (Ref. 8). This assumption implies a high resultant bolt load, R, in the inboard row of bolts.

From experimental analysis, it was known that layups of all ±45 degree laminae would have a superior bolt load

bearing capacity (Ref. 9). Since the wing skin would be required to carry no applied load beyond the inboard row of bolts, it was decided to take advantage of this high bearing load capacity for all designs by requiring that, at the inboard row of bolts, the skin be composed of 100 per cent ±45 degree plies.

Considering a four-hole-diameters-wide column of bolts, the total tensile load L in the skin is given by

$$(1) \quad L = N_x 4D$$

where D is the bolt diameter used in the joint. If there are N bolts in each column,

$$(2) \quad P = \frac{L}{N} = \frac{N_x 4D}{N}$$

As explained in Ref. 10, the bearing stresses in a plate due to bolt loads are a function of the magnitude of the bolt load, the diameter of the bolt hole, and the effective thickness of the plate. For this study t^*, the effective bolt bearing thickness of the plate, is defined as follows:

for $t \leq 2D$, $t^* = t$

for $t > 2D$, $t^* = 2D$

where t is the plate thickness. This definition of effective bearing plate thickness is adopted to account for the fact that in thick plates loaded through a bolt hole the bolt loads tend to distribute themselves so that higher portions of the load are carried at the plate edges than at the plate center. The thickness definition was not chosen

through rigorous experimental or analytical processes but rather in light of engineering experience with metal plates. The bolt bearing stress, σ_{BR}, is

$$(3) \qquad \sigma_{BR} = \frac{\text{Bolt Load}}{D\ t^*}$$

On all but the inboard row of bolts,

$$(4) \qquad \sigma_{bx} = \frac{4\ N_x}{N\ t^*}$$

Since the inboard bolts in the non-buffer strip joints are assumed to react the shear as well as a share of the tensile load, they carry a bolt load, R, given by:

$$(5) \qquad R^2 = \left(\frac{N_x\ 4D}{N}\right)^2 + (N_{xy} 4D\ )^2$$

$$(6) \qquad R = 4DN_x \left[\left(\frac{1}{N}\right)^2 + \left(\frac{N_{xy}}{N_x}\right)^2 \right]^{\frac{1}{2}}$$

The bearing stress on these inboard bolts is

$$(7) \qquad \sigma_{BR} = \frac{4N_x}{t^*} \left[\left(\frac{1}{N}\right)^2 + \left(\frac{N_{xy}}{N_x}\right)^2 \right]^{\frac{1}{2}}$$

Test specimens composed of NARMCO 5208/T300 $[\pm 45]$ layers were found to be able to withstand

$$\sigma_{BR\max} = 78000 \text{ lbf./in.}^2$$

when this load was applied through untorqued bolts (Ref. 9). This value is used in Eq. 7 to determine the minimum allowable joint thickness at the inboard bolt holes. In cases where this is a critical design parameter, the geometry illustrated in Fig. 2(a) is used to size the joint. Otherwise, the geometry illustrated in Fig. 2(b) is used.

The bolt hole center locations are numbered from one to N beginning with location 1 at the first or outboard bolt and ending with location N at the last or inboard bolt. To simplify the analysis, other locations in the joints are specified by a parameter η, defined relative to the bolt hole numbers. $\eta = 1$ means at the position of the first bolt hole center, $\eta = 2$ means at the position of the second bolt hole center, and $\eta = N$ means at the position of the inboard bolt hole center. Intermediate positions are defined as follows:

$\eta = 1.5$ means halfway between $\eta = 1$ and $\eta = 2$

$\eta = 2.5$ means halfway between $\eta = 2$ and $\eta = 3$

$\eta = N-.5$ means halfway between $\eta = N-1$ and $\eta = N$

The bypass force, F_{BP}, in the joint is defined as the tensile load passed from station η to station $\eta+1$, thus,

$$F_{BP} = N_x 4D \left(1-\frac{\eta}{N}\right) \qquad (\eta = \text{integer})$$

The bypass stress, σ_{tx}, is defined as

$$\sigma_{tx} = \frac{F_{BP}}{4Dt}$$

The bypass strain ϵ is defined as

$$\epsilon = \frac{\sigma_{tx}}{E_x}$$

where E_x is the tensile modulus of the material.

The strain level halfway between station η and $\eta+1$ is given by

$$(8)\quad \epsilon_{\eta+.5} = \frac{N_x\left(1 - \frac{\eta}{N}\right)}{t_{\eta+.5}\, E_{x_{\eta+.5}}} \qquad (\eta = \text{integer})$$

Both thickness and tensile modulus of the material are allowed to change with location within the joint.

The stress-strain behavior of a zero degree NARMCO 5208/T300 lamina is linear to failure. As reported in Ref. 2, the modulus of elasticity of such a lamina in the direction of the graphite fibers was determined by experiment to be 20.5 X 10^6 lbf./in.^{2}. The stress-strain behavior of a NARMCO 5208/T300 45 degree lamina is not linear. Tests performed in 1974, (Ref. 11), for a NARMCO 5208/T300 45 degree lamina under room temperature dry conditions reported the stress-strain behavior in the form of a secant modulus, which varied with strain level from 2.9 X 10^6 lbf./in.^{2} at zero strain to 1.8 X 10^6 lbf./in.^{2} at a strain level of 13000 micro-in./in. as shown in Fig. 3. (Secant modulus is the slope of a line through a point on the stress-strain curve and the origin.)

As explained in Ref. 3, the strain state for a balanced laminate composed of n layers is described by

$$(9)\quad \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} \\ A_{12} & A_{22} & A_{24} \\ A_{14} & A_{24} & A_{44} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix}$$

where $[N]$ is a vector of applied tensile and shear loads and $[\epsilon]$ is a vector describing the strain state of the plate. The components of the $[A]$ matrix are defined as

follows:

$$(10)\ A_{ij} = \sum_{k=1}^{n} \bar{C}_{ij}^{k}\, t_k$$

where $\bar{C}_{ij}^{k}$ are the elements of the compliance matrix of the k^{th} lamina.

This study is concerned only with balanced design laminates made from laminae of uniform thickness and material composition and oriented at either zero or ±45 degrees to the spanwise direction. For such laminates, it is seen from Eqs. 9 and 10 that the various moduli vary linearly with the per cent of zero degree plies in the laminate. Because the ±45 degree data on secant modulus was available, and because the secant modulus is convenient for design use, a laminate value of secant modulus was calculated. Figure 4 shows the variation of secant modulus with laminate composition for NARMCO 5208/T300 $[0/\pm45]$ material at $\epsilon = 3000$ micro-in./in. It was prepared assuming that this modulus varies linearly between the experimentally determined values for such laminates with zero and 100 per cent zero degree plies. Figure 4 is used to determine the tension modulus of the various NARMCO 5208/T300 $[0/\pm45]$ laminates used in the study joint designs.

The laminate composition at station $\eta = 1.5$ is assumed to be the same as that of the wing skin outboard of the joint. It determines the tensile modulus at this position. Equation 8 is then used to determine $t_{1.5}$.

The laminate composition at station $\eta = N-.5$ is initially

determined by assuming that the percentage of zero degree plies varies linearly from station $\eta = 1.5$ to station $\eta = N$ where the laminate is to be composed of 100 per cent ±45 degree plies. An additional constraint, applicable only to the non-buffer strip joint, is that the laminate at station $\eta = N-.5$ can have no fewer than 5 per cent zero degree plies. This is done to ensure that there are sufficient load bearing zero degree plies to carry the bypass load between the second to last and the last, or inboard, row of bolts.

Having fixed the laminate composition, the tensile modulus and, through Eq. 8, the thickness at station N-.5 are determined.

The remaining joint thicknesses are determined assuming a linear variation of joint thickness between $\eta = 1.5$ and $\eta = N-.5$. Laminate compositions midway between each pair of adjacent bolts are determined using the thickness distribution and the desired strain level.

$$(11) \quad E_{x_{\eta+.5}} = \frac{N_x \left(1 - \frac{\eta}{N}\right)}{t_{\eta+.5}\,\epsilon} \qquad (\eta = \text{integer})$$

The tension modulus, E_x, of the material is assumed to vary linearly with distance between the values determined from Eq. 11. Thus,

$$(12) \quad E_{x_\eta} = \frac{E_{x_{\eta-.5}} + E_{x_{\eta+.5}}}{2} \qquad (\eta = \text{integer})$$

These modulus values determine the laminate composition throughout the joint.

Joints are sized using 0.25, 0.375, 0.4375, and 0.5 inch bolts with laminate compositions at the first bolt varying from 60 to 10 per cent zero degree plies. These laminate compositions cover the range over which the Eisenmann strength model to be described in the next section was considered accurate. In all cases, the same interbolt strain level, 3000 micro-inches per inch, is maintained. The minimum number of bolts used in any joint design is three. This is done to provide a mechanism by which the desired strain level can be maintained. The maximum number of bolts in any joint is determined by the desired interbolt spacing of four bolt hole diameters and the requirement that the joint length not exceed ten inches.

B. DETERMINING THE STRENGTH OF A NON-BUFFER STRIP JOINT

Waddoups, Eisenmann, and Kaminski, in Ref. 12, showed experimentally that graphite-epoxy laminates are statically brittle and exhibit many of the failure characteristics of brittle materials first explained by Griffith in Ref. 13. They formulated a model which assumed that crack growth behavior in graphite-epoxy laminates was a function of stress intensity and critical energy level, and they verified their model by experiment.

In Ref. 14, Eisenmann continued this work and developed a bolted joint strength model for composite materials. This model accounted for the material ultimate strengths, local stress intensity factors, and geometric width correction factors. With the Eisenmann model, it was

possible to calculate the total stress at any point on a loaded circular hole in an orthotropic plate by linearly combining the various stresses acting upon the plate using Eq. 13. Because of the internal curve fitting techniques used in this model, it was considered to give accurate results for laminate compositions varying from 10 to 60 per cent zero degree plies (Ref. 14).

$$(13)\quad \sigma^i = \lambda^i_{tx} f(\tfrac{a}{r})^i_{tx}\sigma_{tx} + \lambda^i_{ty} f(\tfrac{a}{r})^i_{ty}\sigma_{ty} + \lambda^i_{xy} f(\tfrac{a}{r})^i_{xy}\sigma_{xy} + \lambda^i_{bx} f(\tfrac{a}{r})^i_{bx}\sigma_{bx} + \lambda^i_{by} f(\tfrac{a}{r})^i_{by}\sigma_{by}$$

where:

i = indicator of the exact position on the hole.

λ^i_α = finite width correction factor at location i for applied load condition α

$f(\frac{a}{r})^i_\alpha$ = effective isotropic stress concentration factor at location i for applied load condition α

σ_α = applied stress for load condition α

σ^i = net tangential stress at location i

α = subscript denoting applied load condition as follows:

xy = shear

tx = tension in x - direction

ty = tension in y - direction

bx = bearing in x - direction

by = bearing in y - direction

The stress definitions are sketched in Fig. 5. The λ and f factors are determined by plate geometry and material composition.

The failure modes of NARMCO 5208/T300 $[0/\pm 45]$ plates with loaded holes were determined by test (Ref. 14). Crack initiation in the test specimens most often occurred on the hole edge at positions β=0, ±45, ±90, ±135, or 180 degrees measured from the X axis. From symmetry it was determined that all these failure modes could be adequately described by description of the failure modes encountered at β =0, 45, and 90 degrees. Laminate strength for each of these positions was determined by experiment (Ref. 14). In these tests the bolts used to load the holes were untorqued.

The Eisenmann static strength model was used to calculate stress intensity and geometric width correction factors based upon an interbolt spacing of four hole diameters. Equation 13 was used to prepare Figs. 6-29 which define the failure modes expected for laminates whose composition varies from 60 to 10 per cent zero degree plies with 0.25, 0.375, 0.4375, and 0.50 inch holes. Only the effects of σ_{xy}, σ_{tx}, and σ_{bx} were considered in the preparation of these curves.

One of the parameters of interest in this study is the excess bearing capacity of each joint design. For purposes of this study the excess bearing capacity of the joint is defined as the smallest excess bearing capacity at any

bolt hole in the joint. At each hole a representative stress load, S_1, which accounts for the combined bearing and bypass stresses is calculated as follows:

$$(14) \quad S_1 = \left[\sigma^2_{bx} + \sigma^2_{tx} \right]^{\frac{1}{2}} \quad \text{as loaded}$$

A representative ultimate strength, S_2, is calculated from:

$$(15) \quad S_2 = \left[\sigma^2_{bx} + \sigma^2_{tx} \right]^{\frac{1}{2}} \quad \text{at failure}$$

where the failure state is the state at which

$$(16) \left[\frac{\sigma_{bx}}{\sigma_{tx}} \right]_{\text{failure}} = \left[\frac{\sigma_{bx}}{\sigma_{tx}} \right]_{\text{loaded}}$$

Then, as shown in Fig. 30, the excess bearing capacity, B is defined as

$$(17) \quad B = \frac{S_2 - S_1}{S_1}$$

$B > 0$ at all holes in a joint implies that there is some margin of safety.

$B = 0$ at any hole in a joint implies that there is no margin of safety.

$B < 0$ at any hole in a joint implies that the joint would fail under the applied load.

The effect of laminate composition and bolt hole size on excess bearing capacity is shown in Figs. 31, 32, and 33.

C. DETERMINING THE WEIGHT OF A NON-BUFFER STRIP JOINT

The weights per inch of chord for the non-buffer strip joints were calculated by multiplying the cross-sectional

areas of each joint by the density of NARMCO 5208/T300 graphite-epoxy laminate material. The effect of laminate composition, hole size, and number of bolt holes upon joint weight is shown in Figs. 34, 35, and 36.

III. ANALYSIS OF THE BUFFER STRIP JOINT

A. SIZING THE BUFFER STRIP JOINT

Figure 37 is a schematic of the buffer strip joint design. All bolt holes are of the same diameter. Except for the inboard row, all bolt holes are placed in the buffer strips. This placement was chosen for two reasons:

1. It took advantage of the high bearing capacity of NARMCO 5208/T300 $[\pm 45]$ laminates.
2. It reduced stress concentrations in the heavily loaded primary strips.

The bolt holes in the buffer strips are spaced so that there is a distance of four hole diameters between adjacent hole centers. Two bolt holes are placed in each primary strip which is located between two buffer strips. These bolt holes are placed in a row with the inboard bolt in the buffer strip. In the joint analysis, wing taper is disregarded and the spanwise edges of the buffer and primary strips are considered parallel. It is assumed that joint thicknesses at any position are the same in the buffer and primary strips.

The buffer strips used in the joints analyzed in this study are four hole diameters wide, and the two primary

strips are each 3.335 hole diameters wide. This gives an overall buffer strip joint width of 10.67 hole diameters. These dimensions were chosen because it was felt that they were representative of a geometry which could be used in an advanced fighter type aircraft wing skin application. No attempt is made to justify these dimensions either analytically or experimentally, and no attempt is made to assess the effect of this choice of dimensions upon joint efficiency.

In the following theoretical development, as in the case of the non-buffer strip joint, a tensile load N_x and a shear load N_{xy} are assumed constant across the outboard edge of the joint. It is also assumed that fittings were designed so that each row of bolts in the joint transfers the same portion P of the applied tensile load and so that the shear load is reacted by the inboard row of bolts, each of the inboard bolts carrying an equal share of the shear load as well as a share of the tensile load. To utilize the high bearing capacity of a NARMCO 5208/T300 $\left[\pm 45\right]$ laminate, it was decided to impose a requirement that at the inboard row of bolts, the primary strips be composed of 100 per cent ±45 degree plies.

Considering the joint sketched in Fig. 37, the load, L, on a single buffer strip joint, is given by:

$$(18) \quad L = N_x AD$$

where AD is the overall joint width. The bolt load in all bolts except those in the inboard row is given by

$$(19) \quad P = \frac{L}{M} = \frac{N_x AD}{M}$$

where M is the number of rows of bolts in the joint.

Since the shear load N_{xy} and a total tensile bolt load P are reacted equally by each of the three bolts in the inboard row of bolts, the resultant load, R, on each of these bolts is calculated from

$$(20) \quad R^2 = \left[\frac{N_x AD}{3(M)}\right]^2 + \left[\frac{N_{xy} AD}{3}\right]^2$$

$$(21) \quad R = \frac{ADN_x}{3}\left[\left[\frac{1}{M}\right]^2 + \left[\frac{N_{xy}}{N_x}\right]^2\right]^{\frac{1}{2}}$$

As in the non-buffer strip joint design, the desired interbolt strain level, 3000 micro-inches per inch, is the primary consideration determining the joint geometry for the buffer strip design. Locations in the joint are described by station numbers, just as in the non-buffer strip design. In the case of the buffer strip design, however, the station numbers vary from $\eta=1$, which corresponds to the location of the outboard bolt-hole center, to station $\eta=M$ which corresponds to the location of the row of centers of the inboard bolts.

The thickness at station $\eta=1.5$ is determined by the applied load, the joint geometry, the joint material composition, and the desired interbolt strain level. The average modulus of the joint, $E_{x_{ave}}$, as defined in Ref. 7, is used in calculating this thickness.

$$(22)\quad E_{x_{ave}} = \frac{D(A-W_B)\,E_{x_1} + W_B D E_{x_2}}{AD}$$

where $W_B D$ = width of the buffer strip

E_{x_1} = tensile modulus of the primary strip

E_{x_2} = tensile modulus of the buffer strip

As shown in Fig. 4, E_{x_1} is determined by the percentage of zero degree plies in the primary laminate. As shown in Fig. 3, E_{x_2} varies with strain level.

The thickness at station $\eta = 1.5$ is derived from

$$(23)\quad \epsilon = \frac{ADN_x \left[1-\frac{1}{M}\right]}{ADt\, E_{x_{ave}}}$$

where ϵ is the desired interbolt strain level. Thus,

$$(24)\quad t_{1.5} = \frac{AN_x \left[1-\frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right]\epsilon}$$

Similarly,

$$(25)\quad t_{M-.5} = \frac{AN_x \left[\frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right]\epsilon}$$

The bolt bearing stress in the inboard row of bolts is given by

$$(26)\quad \sigma_{BR} = \frac{R}{t^*}$$

where t^*, the effective bearing thickness, is defined as in the non-buffer strip joint.

As stated earlier, the bearing strength of NARMCO 5208/T300 $[\pm 45]$ degree laminates is 78000 lbf./in.^{2}. This determines the minimum allowable joint thickness at station

$\eta = M$ for any given load condition.

The laminate composition in the primary strips is initially assumed to vary linearly with distance from the composition at station $\eta = 1.5$, where it is the same as that of the plate outboard of the joint, to 100 per cent ±45 degree plies at station $\eta = M$ where the applied shear loads are reacted.

The joint thickness at station $\eta = M-.5$ is determined by the desired interbolt strain level.

$$(27) \quad t_{M-.5} = \frac{AN_x}{M\left[(A-W_B)\, E_{x_1} + W_B E_{x_2}\right]\epsilon}$$

The remaining joint thicknesses are then determined geometrically from those at $\eta = 1.5$ and $\eta = M-.5$ using the same techniques as for the non-buffer strip joint. If possible, a cross-section similar to that shown in Fig. 2(b) is used. When this yields a thickness at the last bolt which is too small, using the maximum bearing stress criteria discussed above and Eq. 26, a cross-section similar to that of Fig. 2(a) is used.

Having fixed the joint geometry, the laminate composition is determined by the requirement that the design strain level be maintained between each pair of bolt holes.

At any station $\eta = k+.5$, $k=1, 2, \ldots, M-1$,

$$(28) \quad \epsilon = \frac{N_x A\left[1-\frac{k}{M}\right]}{\left[E_{x_1}(A-W_B) + E_{x_2} W_B\right] t}$$

Since the composition of the buffer strips is fixed and E_{x_2}

is determined by the interbolt strain level, it is necessary to vary E_{x_1} and hence the composition of the primary strips to maintain the desired strain level as the bypass loads in the joint vary from hole to hole.

$$(29) \quad E_{x_1} = \frac{1}{(A-W_B)} \left[\frac{N_x A \left[1-\frac{k}{M}\right]}{t_\eta \; \epsilon} - W_B E_{x_2} \right]$$

It is assumed that E_{x_1} varies linearly with distance between stations $\eta = k-.5$ and $\eta = k+.5$. Thus

$$(30) \quad E_{x_{1_k}} = \frac{E_{x_{1_{k+.5}}} + E_{x_{1_{k-.5}}}}{2}$$

Determination of $E_{x_{1_k}}$ determines the required percentage of zero degree plies in the primary laminate at station k.

Having determined the joint geometry and laminate composition, it is then possible to determine the joint weight and excess bearing capacity.

B. DETERMINING THE STRENGTH OF A BUFFER STRIP JOINT

The bolt load P is not carried entirely in the buffer strip. Part of it is transmitted, in shear, to the primary strips. This load splitting is shown in Fig. 38 in which P_1 is the portion of the bolt load P reacted through each of the primary strips and P_2 is the portion of P reacted through the buffer strip. The relationship between P_1 and P_2 is determined analytically as follows:

Referring to Fig. 38,

$$(31) \quad P = 2P_1 + P_2$$

The bypass strain levels in the buffer and primary strips

are assumed the same. Defining

$$(32) \quad 2W_1D = AD - W_BD$$

$$(33) \quad 2W_1 = (A - W_B)$$

$$(34) \quad \frac{P_1}{E_{x_1}W_1t} = \frac{P}{2E_{x_1}W_1t + E_{x_2}W_Bt}$$

Rearranging Eq. 34 gives an expression for P_1 in terms of P, the joint composition, and the joint geometry.

$$(35) \quad \frac{P_1}{P} = \frac{E_{x_1}W_1t}{2E_{x_1}W_1t + E_{x_2}W_Bt}$$

This equation is rewritten in the form

$$(36) \quad P_1 = A_nP$$

where A_n is determined by knowledge of the joint laminate composition, geometry, and strain level.

$$(37) \quad A_n = \frac{E_{x_1}W_1}{2E_{x_1}W_1 + E_{x_2}W_B}$$

The bolt load splitting discussed above is dependent upon the ability of the ±45 degree laminae to transfer a shear load, F_S, from the buffer to the primary strips. It was experimentally determined that failure of this load transferring mechanism could be expected when the shear stress in these fibers, f_s, reached a magnitude of 90,000 lbf./in.^{2} (Ref. 9). At any station in the joint

$$(38) \quad F_{S_\eta} = \frac{P_1}{4Dt_{\pm 45}} \qquad (\eta = \text{integer})$$

where $t_{\pm 45}$ is the total thickness of ±45 degree laminae through which P_1 is transferred. Defining Z = percentage of zero degree plies in the primary strips,

$$(39) \quad t_{\pm 45} = t \, \frac{(100-Z)}{100}$$

$$(40) \quad f_{s_\eta} = \frac{N_x \; A \; A_\eta}{4M \; t_\eta \frac{(100-Z)}{100}} \qquad (\eta = \text{integer})$$

The shear stress f_s varies from bolt to bolt in a given buffer strip joint design. In all designs, however, the highest values for f_s occur at the first bolt hole. Thus f_{s_1} determines the upper limit on the percentage of zero degree plies in the primary strip laminate. From Eq. 24,

$$(41) \quad t_1 = \frac{AN_x \left[1-\frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right]\epsilon}$$

Fig. 4 yields the following relationship between laminate composition and modulus,

$$(42) \quad E_{x_1} = 10^6 \left[20.7-(100-Z)\left[\frac{20.7-2.8}{100}\right]\right] (\text{lbf./in.}^2)$$

Then,

$$(43) \quad \frac{100-Z}{100} = \left[20.7 - \frac{E_{x_1}}{10^6}\right] \frac{1}{17.9}$$

Substituting Eqs. 37, 41, and 43 into Eq. 40,

$$(44) \quad f_{s_1} = \frac{W_1 E_{x_1}}{(M-1)} \, \frac{1}{4} \, \frac{\epsilon_1}{\left[20.7 - \frac{E_{x_1}}{10^6}\right]} \, \frac{1}{17.9}$$

From Eq. 44 it is seen that f_{s_1} is determined by the joint

geometry, laminate composition, and design strain level. Fixing the joint geometry and the design strain level determines the maximum allowable modulus for the primary strips and hence provides an upper limit on the per cent zero degree plies which can be used in the primary strips. This limit is determined by setting f_{s_1} equal to its experimentally determined maximum, 90,000 lbf./in.^{2} and using Eq. 45 to determine $E_{x_{1_{max}}}$.

(45)

$$E_{x_{1_{max}}} = \frac{4(M-1)(20.7)f_{s_1}}{17.9(W_1)(\epsilon_1)10^6 + 4(M-1)f_{s_1}} \quad 10^6(\text{lbf./in.}^2)$$

Under the design conditions applicable to this study, Eq. 45 implies an upper limit of 92 to 98 per cent zero degree plies in the primary strips.

Three other failure modes of a buffer strip joint were found to be most probable under N_x and N_{xy} loading (Ref. 15). Type I and Type II failure occurred in the buffer strip at the loaded bolt holes. Type I failure was characterized by radial cracks at 45 degrees to the X axis. Type II failure was characterized by radial cracks originating at the edge of the hole at 90 degrees to the X axis. Type III failure occurred when the load bearing fibers in the primary strips were broken. These three failure modes are sketched in Fig. 39.

The Type I failure mode characterizes the interaction of bearing and shear stresses in the buffer strip. It was found to be essentially independent of the bypass

stress. The load curves describing this failure mode were derived from experimental results (Ref. 15). Single hole specimens of buffer strip joint material were clamped in test machines along either one or two sides as shown in Fig. 40. With the bolts torqued, the specimens were loaded, and the failure bolt load stresses measured. Tests were run for specimens with 0.25 inch and 0.4375 inch diameter holes. For the doubly clamped test cases Type I failure occurred under the following loads:

$$D = 0.250 \text{ in. } \sigma_{bx_{max}} = 151{,}200 \text{ lbf./in.}^2$$

$$D = 0.4375 \text{ in. } \sigma_{bx_{max}} = 144{,}100 \text{ lbf./in.}^2$$

For the test specimens clamped at only one edge, Type I failure occurred under the following loads:

$$D = 0.25 \text{ in. } \sigma_{bx} = 111{,}000 \text{ lbf./in.}^2$$

$$D = 0.4375 \text{ in. } \sigma_{bx} = 107{,}000 \text{ lbf./in.}^2$$

Satisfactory test results for the pure shear load case could not be obtained.

An attempt was made to approximate the shear effects by superposition of the singly and doubly clamped test results. The finite element computer program ISANIS, listed in Appendix A, was used to analyze the stress concentration field in various orthotropic plates. It was found that for a square plate made from uniform material with a central hole and sides at least four hole diameters in length, the stress field at the hole due to pure applied

shear could be closely approximated by an appropriate superposition of the stress fields resulting from singly and doubly clamped load cases. The superposition used is shown schematically in Fig. 41. The key assumption in this superposition is that the moment reaction which is representative of the single clamped edge load case can be replaced by a couple of equal magnitude resulting from shear loads applied on the upper and lower edges of the specimen. This assumption is really an application of the St. Venant principle that points in a body removed from load application locations react to the load applied rather than its mechanism of application. ISANIS was used to test this assumption and, under the conditions stated above, it was found to be reasonable.

The test specimens were thin and hence $t^*=t$. Therefore,

$$(46) \quad P = \sigma_{bx} D t$$

For the double clamped specimen, the reaction force, F_{R_D}, is given by

$$(47) \quad F_{R_D} = \frac{P}{2} = \frac{\sigma_{bx} D t}{2}$$

Assuming that this force is uniformly distributed across the clamped edges, an edge stress is defined

$$(48) \quad \sigma_{R_D} = \frac{F_{R_D}}{\ell D t}$$

where ℓ X D equals the length of the clamped edge.

For the single clamped specimen, the reaction force,

F_{R_S}, is equal to the applied bolt load P. The reaction moment, m, is given by

$$(49) \quad m = P \frac{\ell D}{2}$$

This moment is approximated by a couple of the same magnitude formed by forces acting on the edges of the specimen which are normal to the clamped edge. For square specimens of side length ℓD, the magnitude of these forces, F, is given by

$$(50) \quad F \ell D = m$$

$$(51) \quad F \ell D = \frac{P \ell D}{2}$$

$$(52) \quad F = \frac{P}{2}$$

This development is also shown schematically in Fig. 41.

Assuming that each force F is distributed uniformly along the specimen edge upon which it is applied, an edge shear is defined

$$(53) \quad \sigma_{xy} = \frac{F}{\ell D t}$$

$$(54) \quad \sigma_{xy} = \frac{P}{2 \ell D t}$$

For the test specimens each side was four hole diameters in length. From Eqs. 47 and 54, the superposition yields

$$(55) \quad \sigma_{xy} = \frac{\sigma_{bx}}{8}$$

Under these assumptions, the experimental data listed previously yield the following failure states:

$$D = 0.25 \text{ in.}$$

$\sigma_{xy} = 0$ $\sigma_{bx} = 151{,}200 \text{ lbf./in.}^2$

$\sigma_{xy} = 13{,}875 \text{ lbf./in.}^2$ $\sigma_{bx} = 111{,}000 \text{ lbf./in.}^2$

$$D = 0.4375 \text{ in.}$$

$\sigma_{xy} = 0$ $\sigma_{bx} = 144{,}100 \text{ lbf./in.}^2$

$\sigma_{xy} = 13{,}375 \text{ lbf./in.}^2$ $\sigma_{bx} = 107{,}000 \text{ lbf./in.}^2$

Figure 42 was prepared from these data points assuming that the ultimate bearing stress-shearing stress interaction curve was linear for the graphite-epoxy laminates used in this study.

Buffer strip Type II failure curves were experimentally determined for buffer strip joint specimens with 0.4375 inch bolt holes, a buffer strip width of 1.5 inches, and primary strips each 1.25 inches wide (Ref. 15). The hole centers were spaced four diameters apart. These tests were run with primary strips composed of 30 per cent zero degree plies and 50 per cent zero degree plies with bolt loads applied through torqued bolts. These test specimens were thin enough so that $t^*=t$. Tests were also run on buffer strips alone to determine the ultimate bypass stress of the buffer strip material (Ref. 15). With no applied bearing stress it was found that the ultimate bypass stress in the buffer strip material was 25000 lbf./in.^{2}. The results of this series of tests indicated that the Type II failure mode for these test specimens could be closely described by the empirical relationship

$$(56) \quad \sigma_{\text{net buffer}} = \frac{P_2 + .25(2P_1)}{D(W_B - 1)\ t_2} + \sigma_{tx_{2_{\text{net area}}}}$$

$\sigma_{\text{net buffer}}$ is the ultimate bypass stress in the buffer strip normalized to the width of the buffer strip minus the diameter of the bolt hole. $\sigma_{tx_{2_{\text{net area}}}}$ is the actual bypass stress in the buffer strip normalized to the width of the buffer strip minus the hole diameter. Thus,

$$(57) \quad \sigma_{\text{net buffer}_{\max}} = 25000 \text{ lbf./in.}^2 \left(\frac{W_B}{W_B - 1}\right)$$

$$(58) \quad \sigma_{tx_{2_{\text{net area}}}} = \frac{\sigma_{tx_2} W_B}{(W_B - 1)}$$

Assuming that the strain levels in the buffer and primary strips are the same at any station η,

$$(59) \quad \frac{\sigma_{tx_1}}{E_{x_1}} = \frac{\sigma_{tx_2}}{E_{x_2}} = \epsilon$$

Thus,

$$(60) \quad \sigma_{tx_{2_{\text{net area}}}} = \frac{\sigma_{tx_1}}{E_{x_1}} E_{x_2} \frac{W_B}{(W_B - 1)}$$

$$(61) \quad \sigma_{bx} = \frac{P}{Dt}$$

$$(62)$$

$$25000 \left(\frac{W_B}{W_B - 1}\right) = \frac{\sigma_{bx}(1 - 1.5\ A_n)}{(W_B - 1)} + \frac{\sigma_{tx_1} E_{x_2}}{E_{x_1}} \frac{W_B}{(W_B - 1)}$$

For a fixed laminate composition in the primary strips, different values of σ_{tx_1} produce different strain levels. This implies variation in E_2, the modulus of the buffer

strip material, with bypass stress in the primary strips. This variation in modulus explains why the Type II failure is not linear in σ_{bx} and σ_{tx_1}.

Type III failure is characterized by fracture of the zero degree fibers in the primary strips. As reported in Ref. 2, this failure mode is encountered when the strain level in these strips reaches 10,000 micro-inches per inch. This failure mode is mathematically predicted by considering the presence of both the bypass stress and the stress due to P_1 in the primary strips. Thus,

$$(63) \quad \sigma_{tx_1} + \frac{A_n P}{W_1 D t} = \sigma_1$$

$$(64) \quad \sigma_{1_{ult}} = \epsilon_{ult} E_1$$

$$(65) \quad P = \sigma_{bx} D t^*$$

$$(66) \quad \sigma_{tx_1} + \frac{A_n \sigma_{bx} D t^*}{(A - W_B) D t} = \epsilon_{ult} E_1$$

The results of these tests are summarized in Fig. 43. The Type I failure line is drawn for a zero shear case and is determined from experiment with a double clamped test specimen, the Type II failure lines are drawn from Eq. 62, and the Type III failure lines are drawn from Eq. 66. For the joints used in this study, Eq. 62 becomes

$$(67) \quad \frac{\sigma_{bx}(1-1.5A_n)}{3} + \sigma_{tx_1} \quad \frac{E_{x_2}}{E_{x_1}} \frac{4}{3} = 25 \frac{4}{3}$$

and Eq. 66 becomes

$$(63) \qquad \sigma_{tx_1} + \frac{A_n \sigma_{bx}}{6.67} = \epsilon_{ult} E_1$$

Figures 44 and 45 describe the expected failure states for the buffer strips used in the study joints. The Type I failure lines on these figures are taken from the zero shear ultimate bearing stresses indicated on Fig. 42. The Type II and Type III failure lines shown in these figures are drawn from Eqs. 67 and 68.

Excess bearing capacity calculations were made for buffer strip joints just as had been done for non-buffer strip joints. In the case of the buffer strip joints, however, only 0.25 and 0.4375 inch holes were considered since Type I failure test data was available only for these hole sizes. The results of these calculations are presented in Figs. 46 and 47.

C. DETERMINING THE WEIGHT OF A BUFFER STRIP JOINT

The weight per inch of chord of each buffer strip design was calculated just as had been done for the non-buffer strip joints by multiplying the cross-sectional area of each joint by the density of the NARMCO 5208/T300 graphite-epoxy material used in the study. The variation of joint weight with primary strip laminate composition, bolt hole size, and number of bolts is shown in Figs. 48 and 49.

IV. DISCUSSION OF RESULTS

A. NON-BUFFER STRIP JOINTS

The following generalizations about non-buffer strip joints were found to be valid:

1. For a given hole size, the joint weight decreased as the percentage of zero-degree plies in the joint increased.
2. For a given hole size, the fewer the number of bolt holes per unit chord, the lighter the joint.
3. The smaller the bolt holes, the lighter the joint could be made.
4. The smaller the bolt holes, the greater the range in number of allowable bolt holes per unit chord.
5. For a given laminate composition, the smaller the bolt holes the larger the minimum number of bolt holes per unit chord required.
6. The smaller the bolt holes, the larger the allowable range of laminate composition.
7. For a given laminate composition, the smaller the bolt holes, the larger the excess bearing capacity.

B. BUFFER STRIP JOINTS

Observations 1-6, above, are also true for buffer strip joints. The excess bearing capacity of the buffer

strip joint, as seen in Figs. 48 and 49, is, in a gross sense, independent of hole size and more heavily influenced by laminate composition, the number of holes, and the load condition.

C. COMPARISON OF NON-BUFFER STRIP AND BUFFER STRIP JOINTS

In Ref. 16 it is explained that the bearing strength of bolted plates is higher when the bearing loads are applied through torqued bolts than when they are applied through untorqued bolts. Since the failure modes of the buffer strip joints were derived from experiments in which the bolt loads were applied through torqued bolts, and since the failure modes of the non-buffer strip joints were derived from experiments in which the bearing loads were applied through untorqued bolts, this may explain at least part of the apparently higher excess bearing capacities available with buffer strip joints. Buffer strip joints generally weigh more than non-buffer strip joints. The fact that buffer strip joints require fewer bolts per inch of chord than non-buffer strip joints can be used to offset some of this weight difference if bolt weights are included in the net joint weight. The reduced number of bolts per inch of chord possible with buffer strip joints should also reduce joint fabrication costs by reducing the number of drilling operations required. Buffer strip joints can be constructed for a larger range of laminate composition than non-buffer strip joints.

V. CONCLUSIONS AND RECOMMENDATIONS

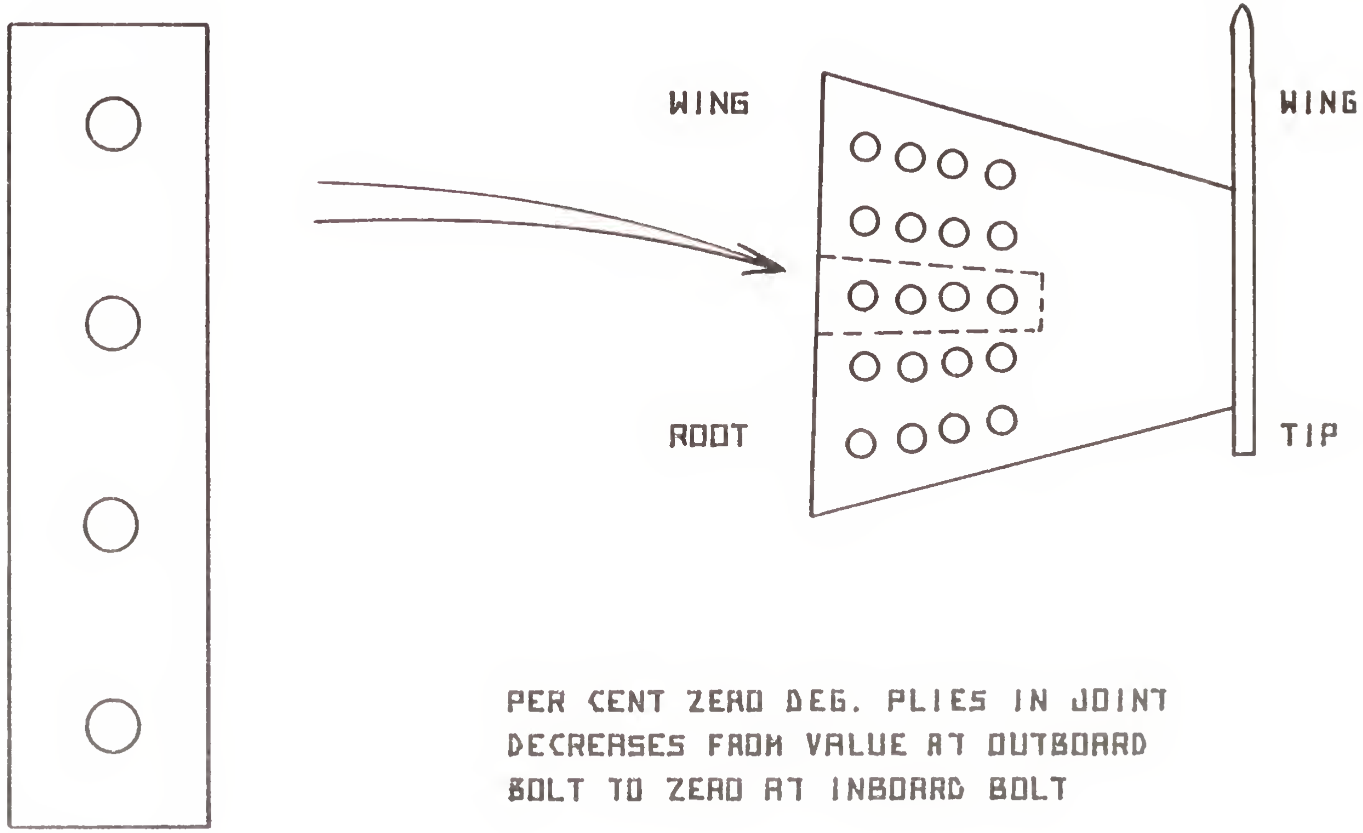
The design methodology used in this study is shown to be capable of producing workable joint designs. The wide range of weights and excess bearing capacities exhibited by the different joint designs indicates that the joint efficiencies achieved by geometric substitution of advanced composites for conventional structural materials were probably minimal when compared with those which would be achieved by designs which took advantage of the special high strength and high modulus properties of advanced composites.

Under the design conditions adopted in this study, buffer strip joints were found to be stronger and more cheaply manufactured than non-buffer strip joints. These advantages are offset by the increased joint weights characteristic of buffer strip joints. In spite of this weight penalty, it is felt that the high excess bearing capacity and integral crack stoppage capability of buffer strip joints makes them promising candidates for aerospace applications.

It is felt that Figs. 31-36 and 46-49 can be used to compare various design proposals and to estimate the costs of variation in laminate composition, hole size, or number of holes. No attempt was made to determine the effect of the design limitations summarized in TABLE I which were placed upon allowable joint geometry and

composition by such factors as manufacturing considerations and fitting interface requirements. It is recommended that the effect of these restrictions be determined by an analysis similar to this one with the restrictions removed.

It is recommended that application of the buffer strip technique to critical components be preceded by further experimentation to more accurately determine the behavior of buffer strip joints under shear loads.



PER CENT ZERO DEG. PLIES IN JOINT DECREASES FROM VALUE AT OUTBOARD BOLT TO ZERO AT INBOARD BOLT

FIGURE 1. SCHEMATIC OF A WING WITH A NON-BUFFER STRIP JOINT

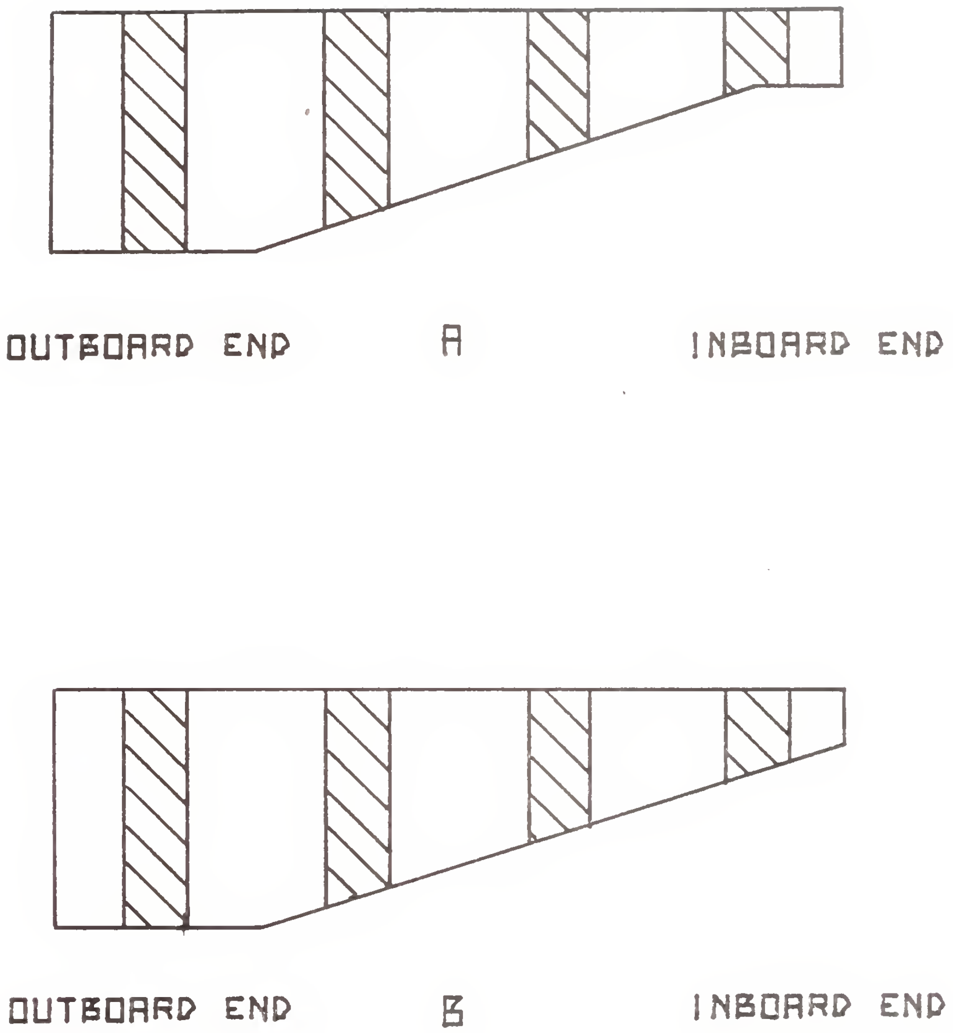


FIGURE 2. PERMISSIBLE JOINT CROSS SECTIONS

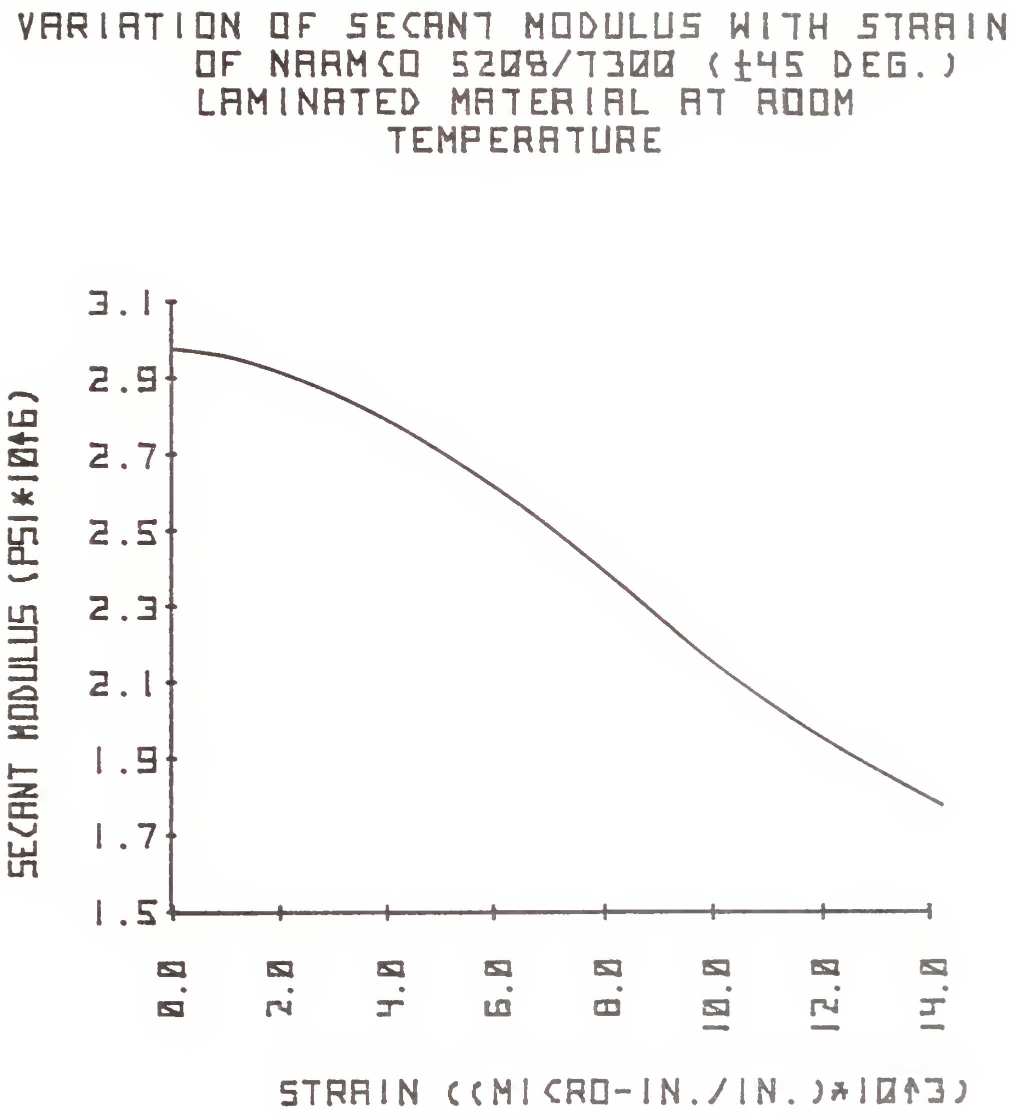


FIGURE 3. VARIATION OF SECANT MODULUS WITH STRAIN OF NARMCO 5208/T300 (±45 DEG.) LAMINATED MATERIAL AT ROOM TEMPERATURE

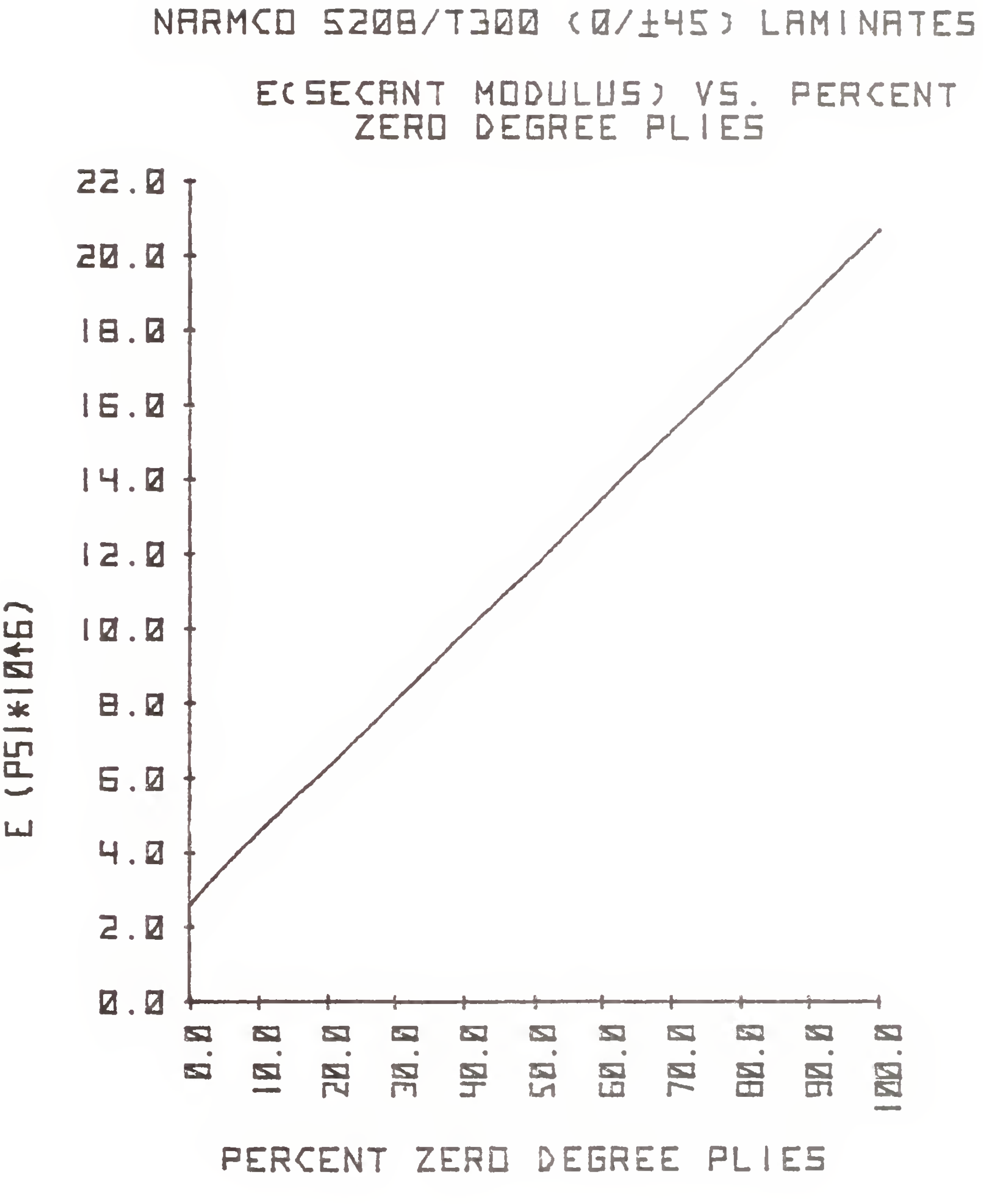


FIGURE 4. SECANT MODULUS OF NARMCO 5208/T300 $[0/\pm 45]$ LAMINATES VS. PER CENT ZERO DEGREE PLIES

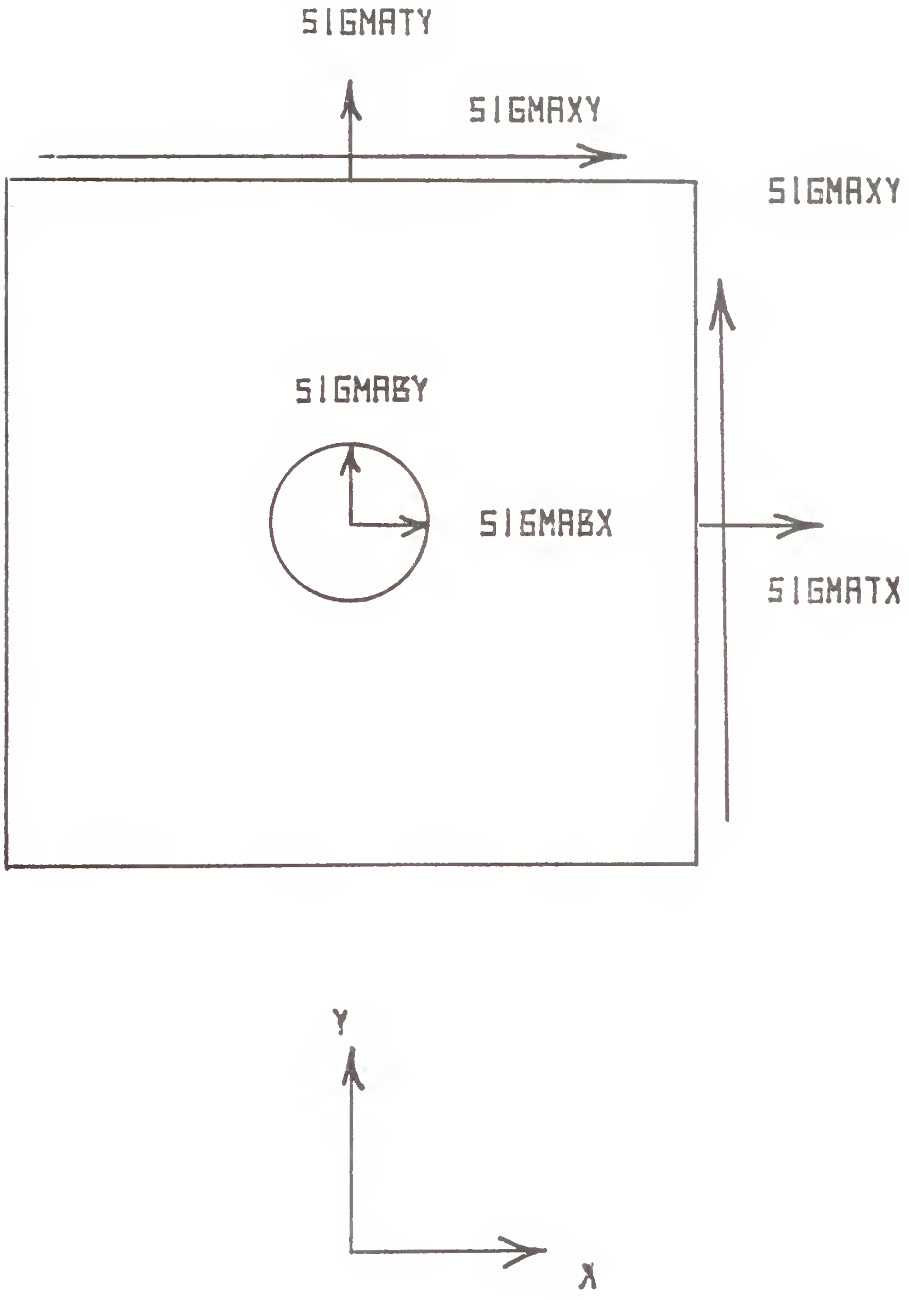


FIGURE 5. BOLTED JOINT APPLIED STRESS DEFINITIONS

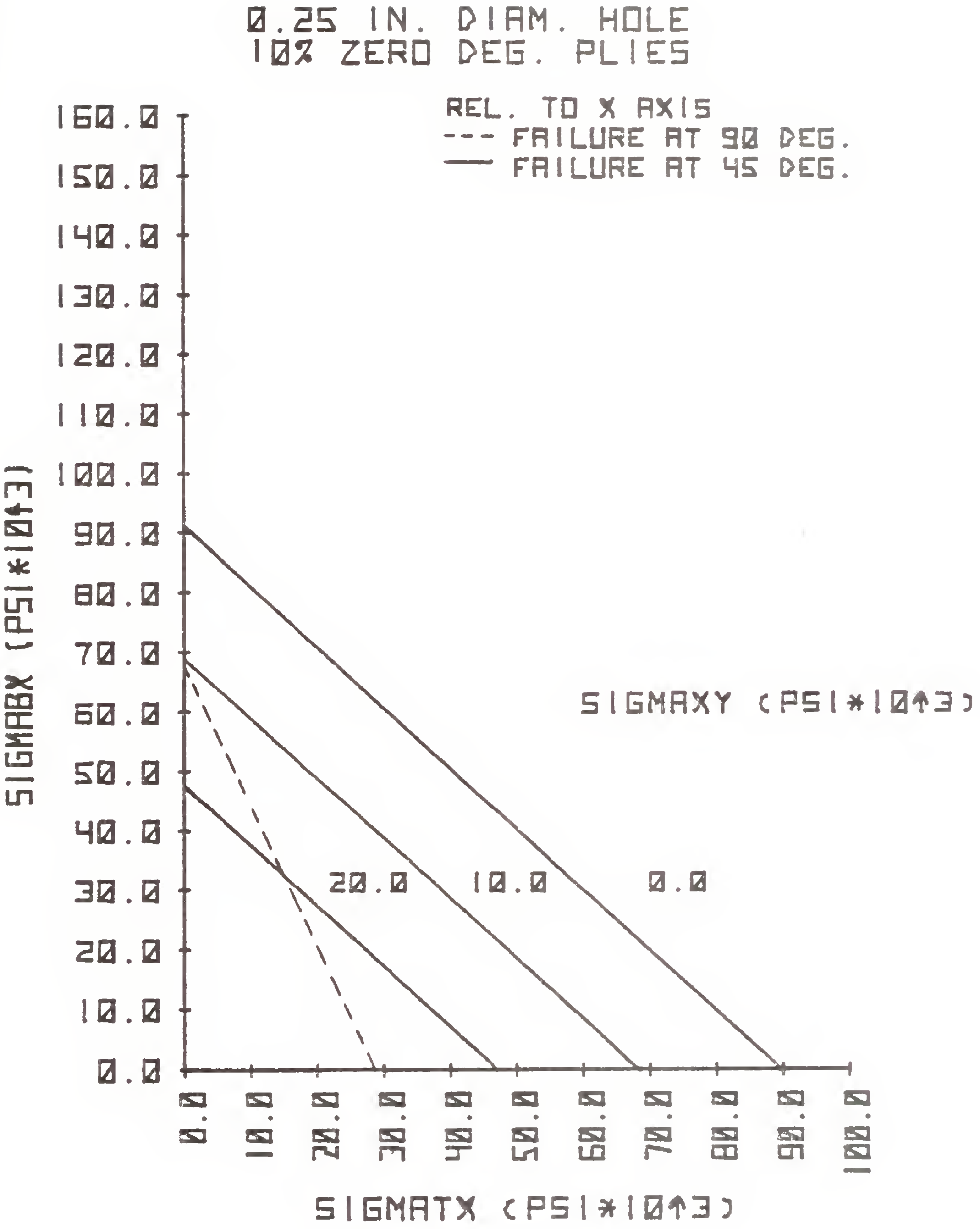


FIGURE 6. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

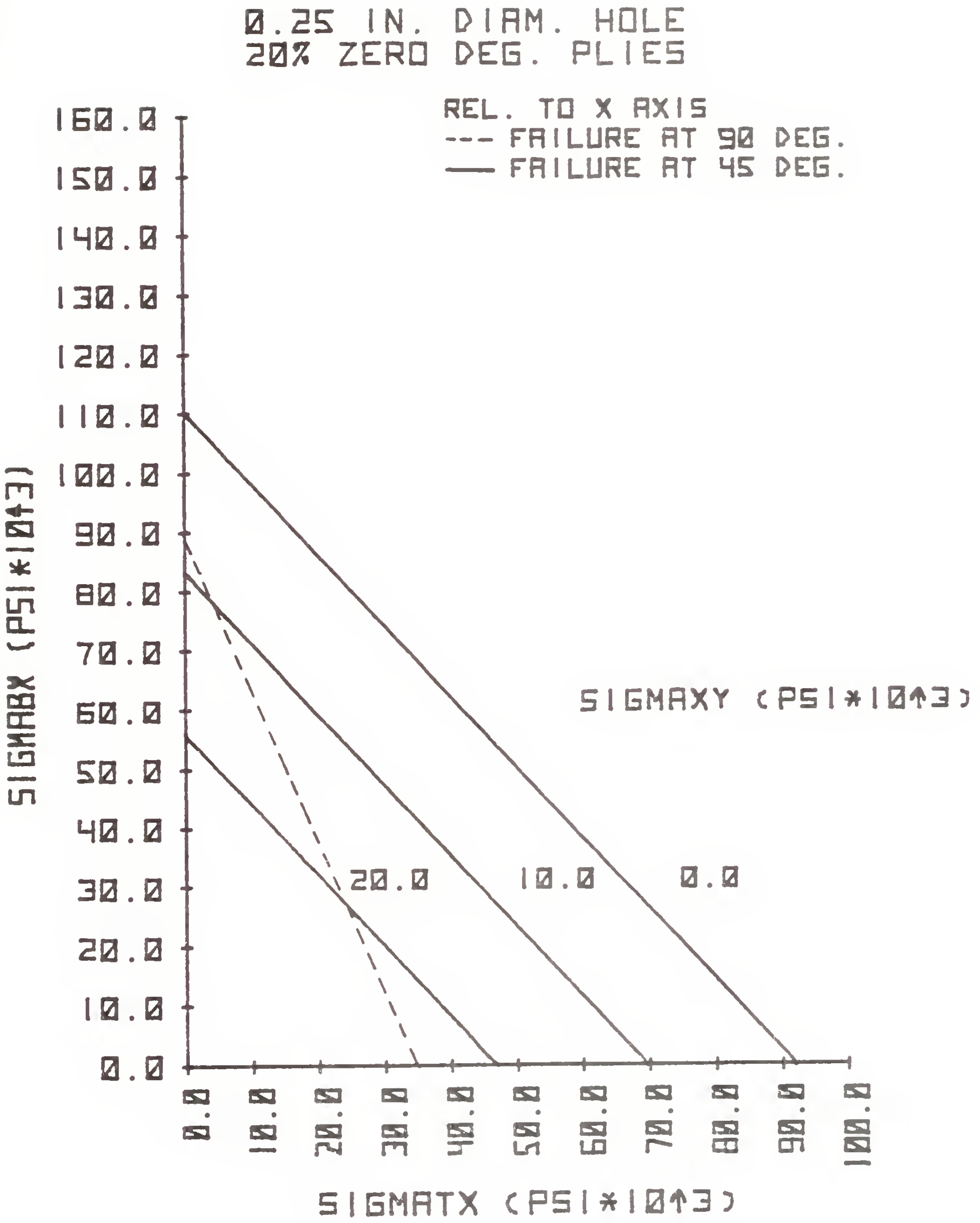


FIGURE 7. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

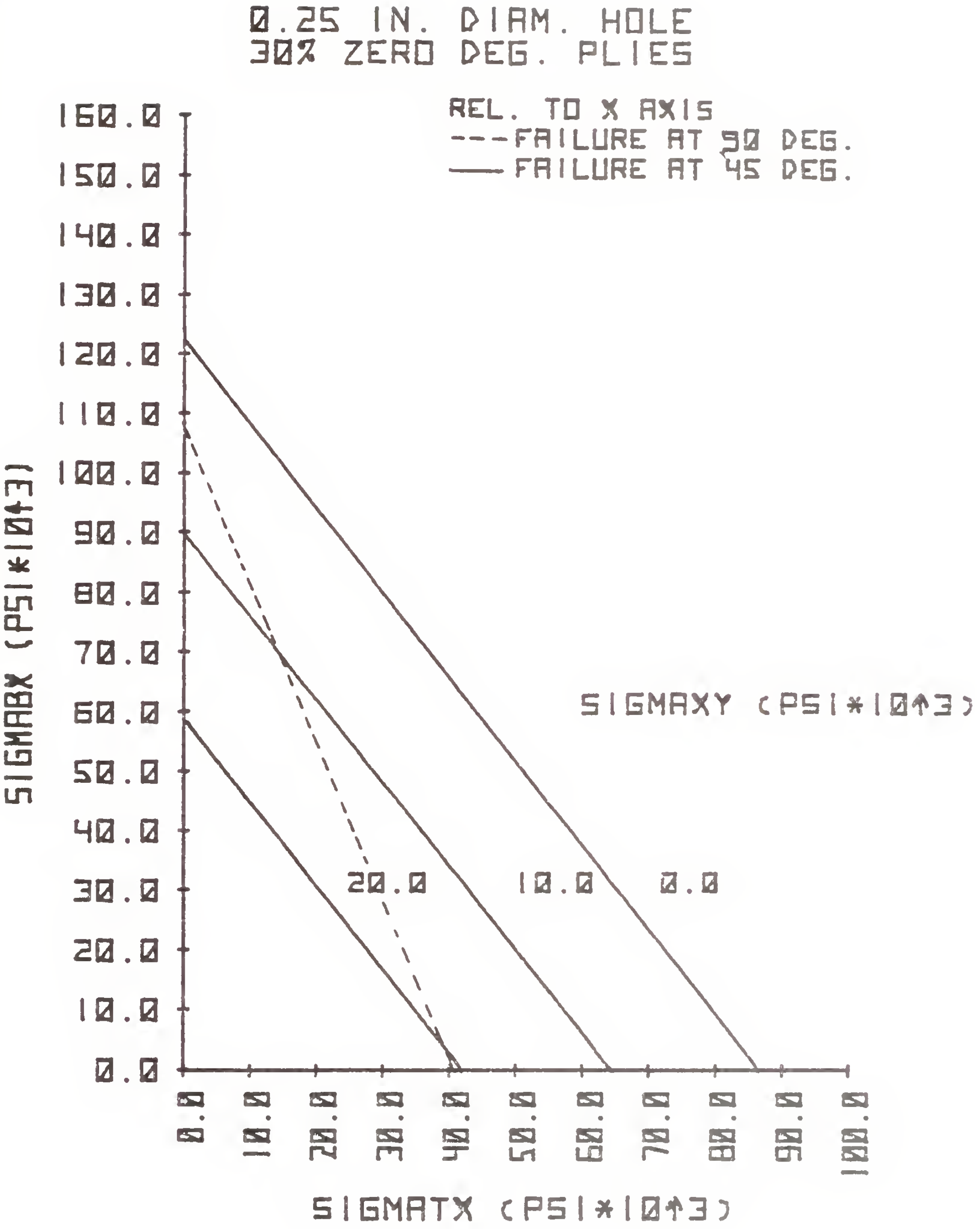


FIGURE 8. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

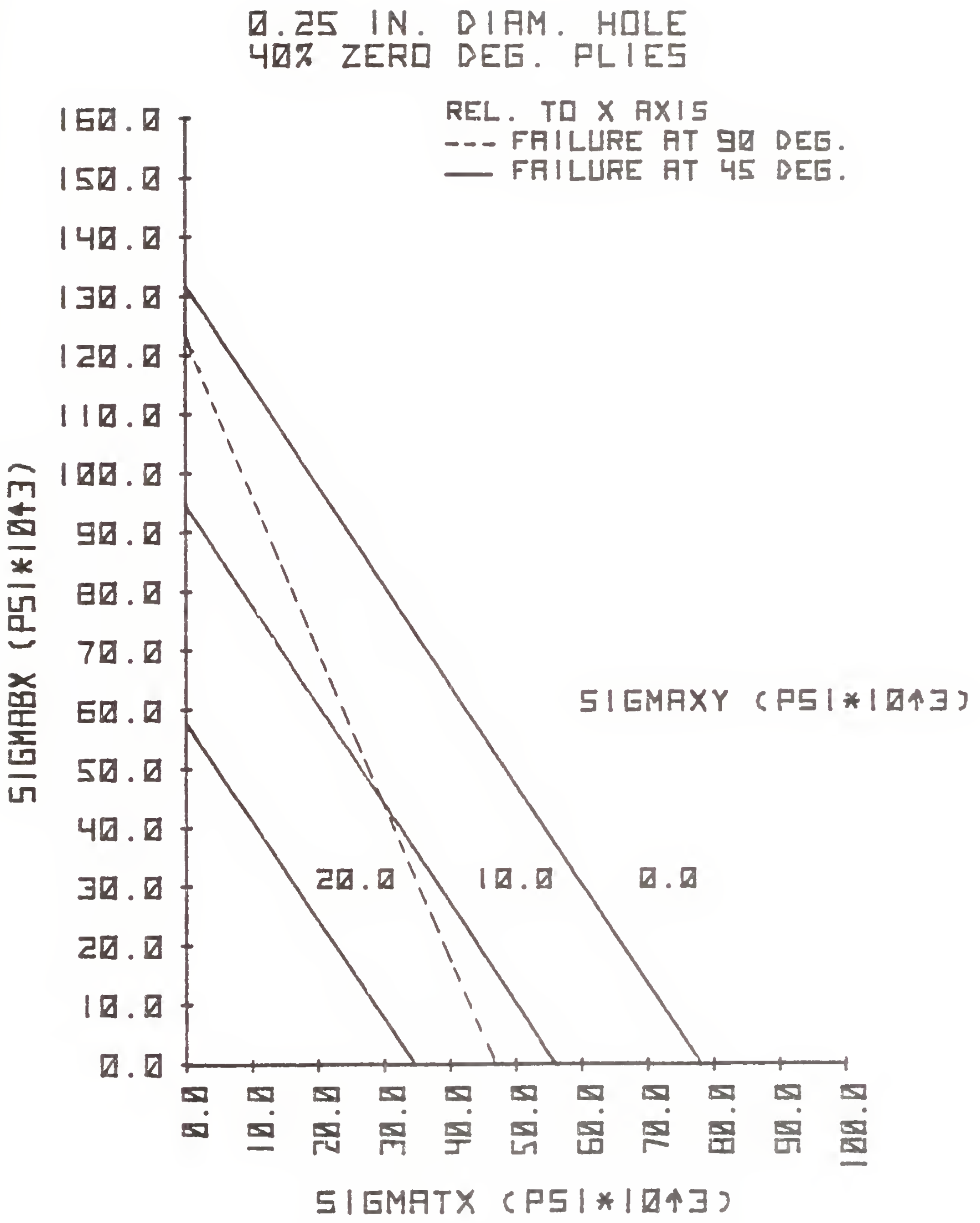


FIGURE 9. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

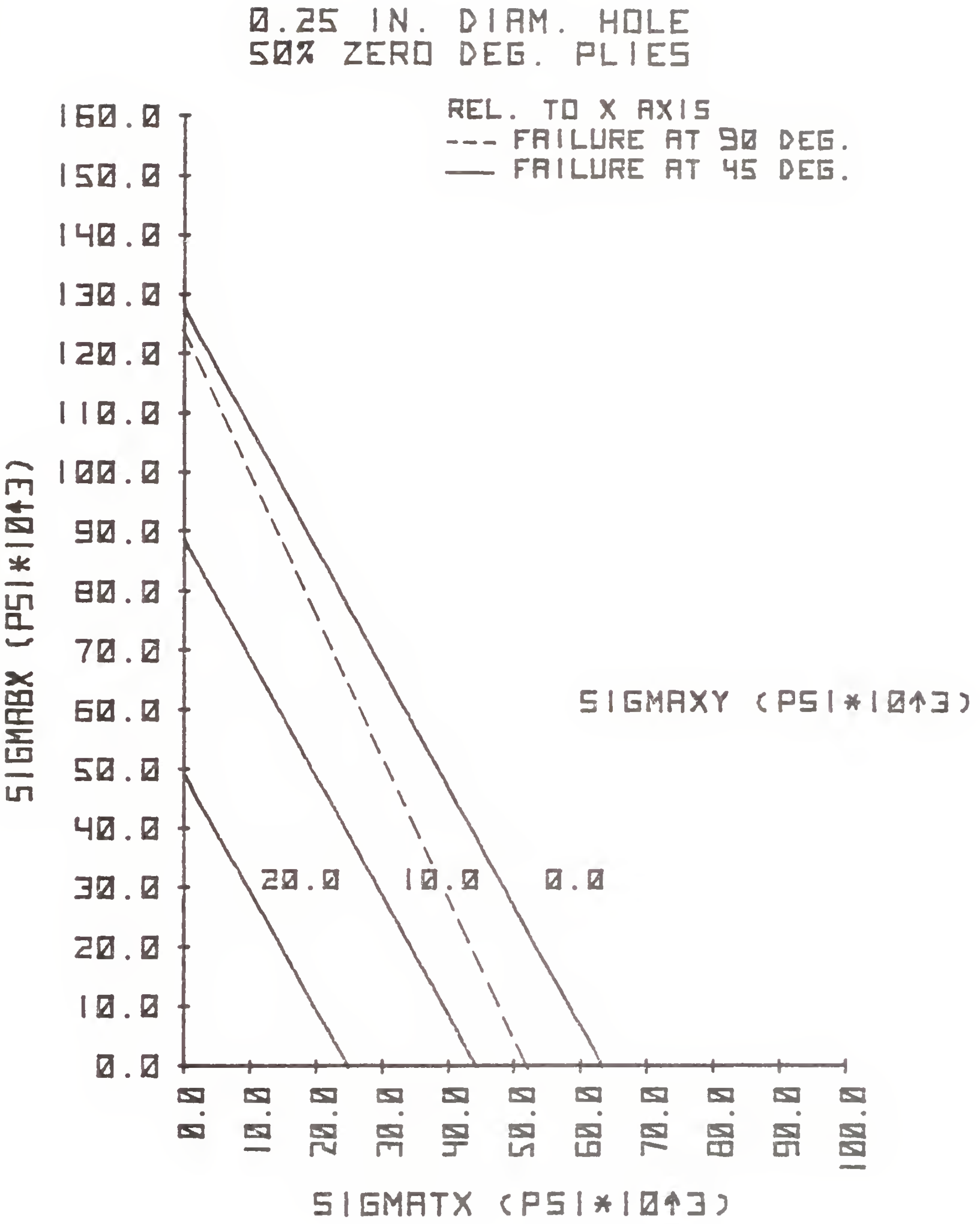


FIGURE 10. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

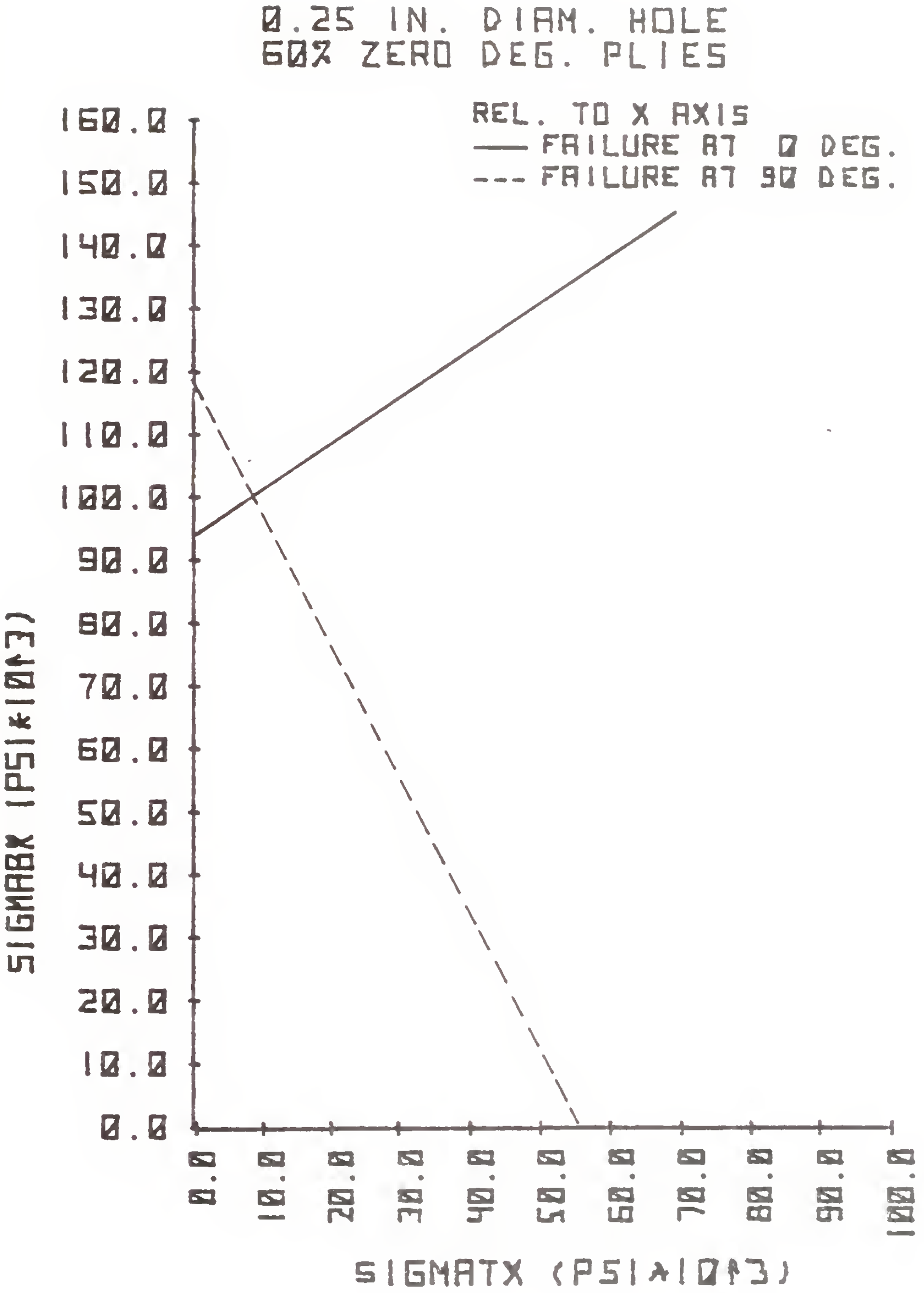


FIGURE 11. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

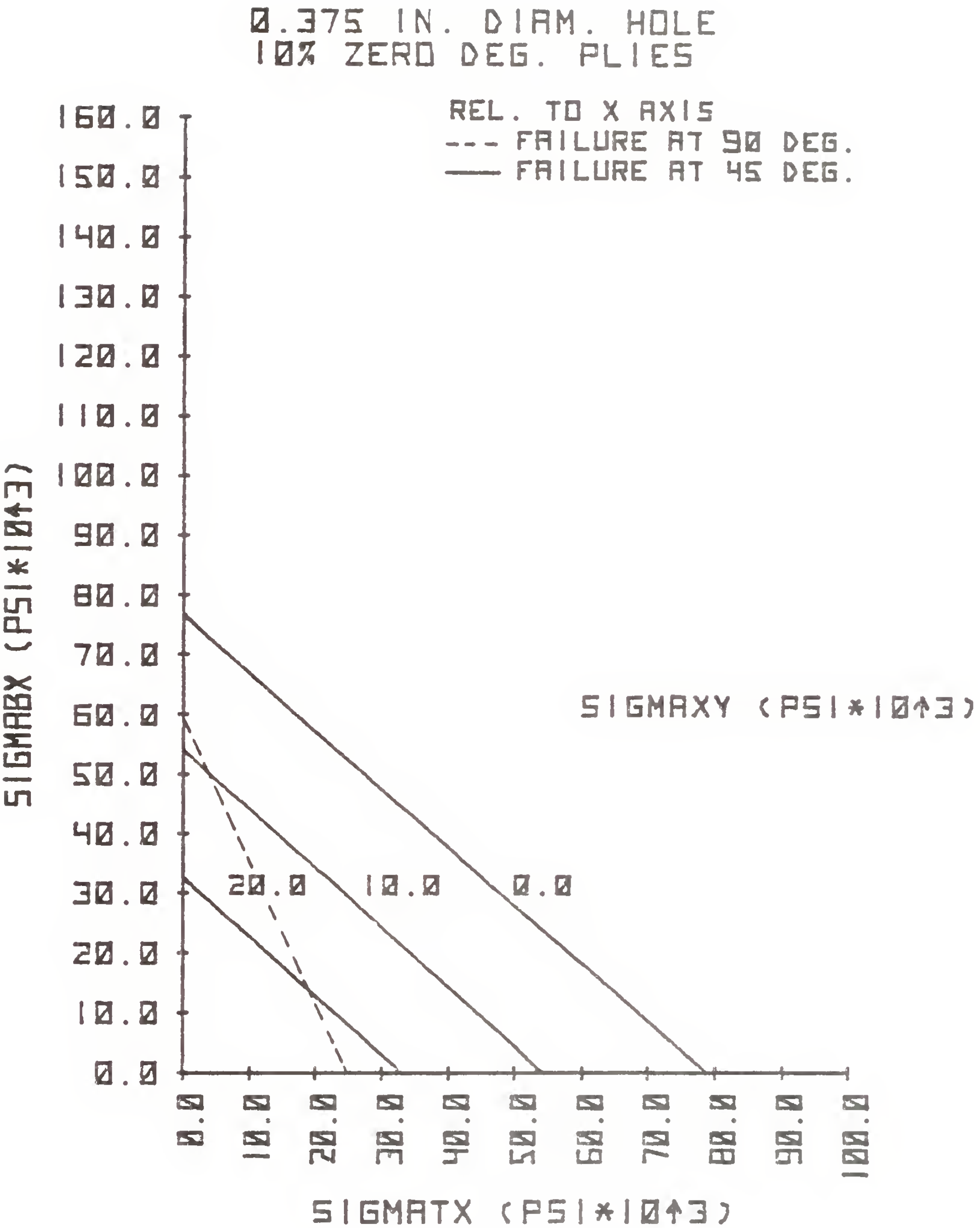


FIGURE 12. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

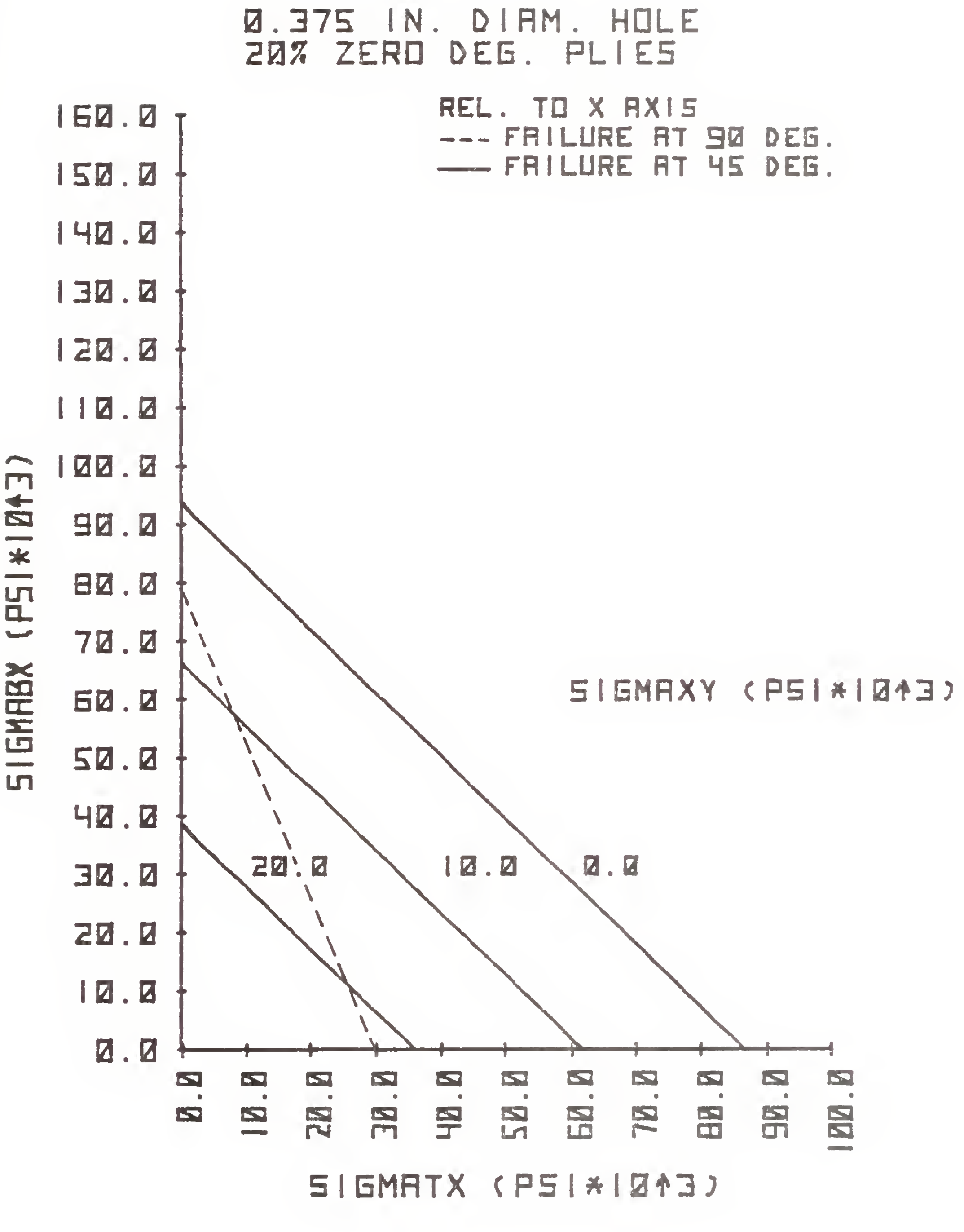


FIGURE 13. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

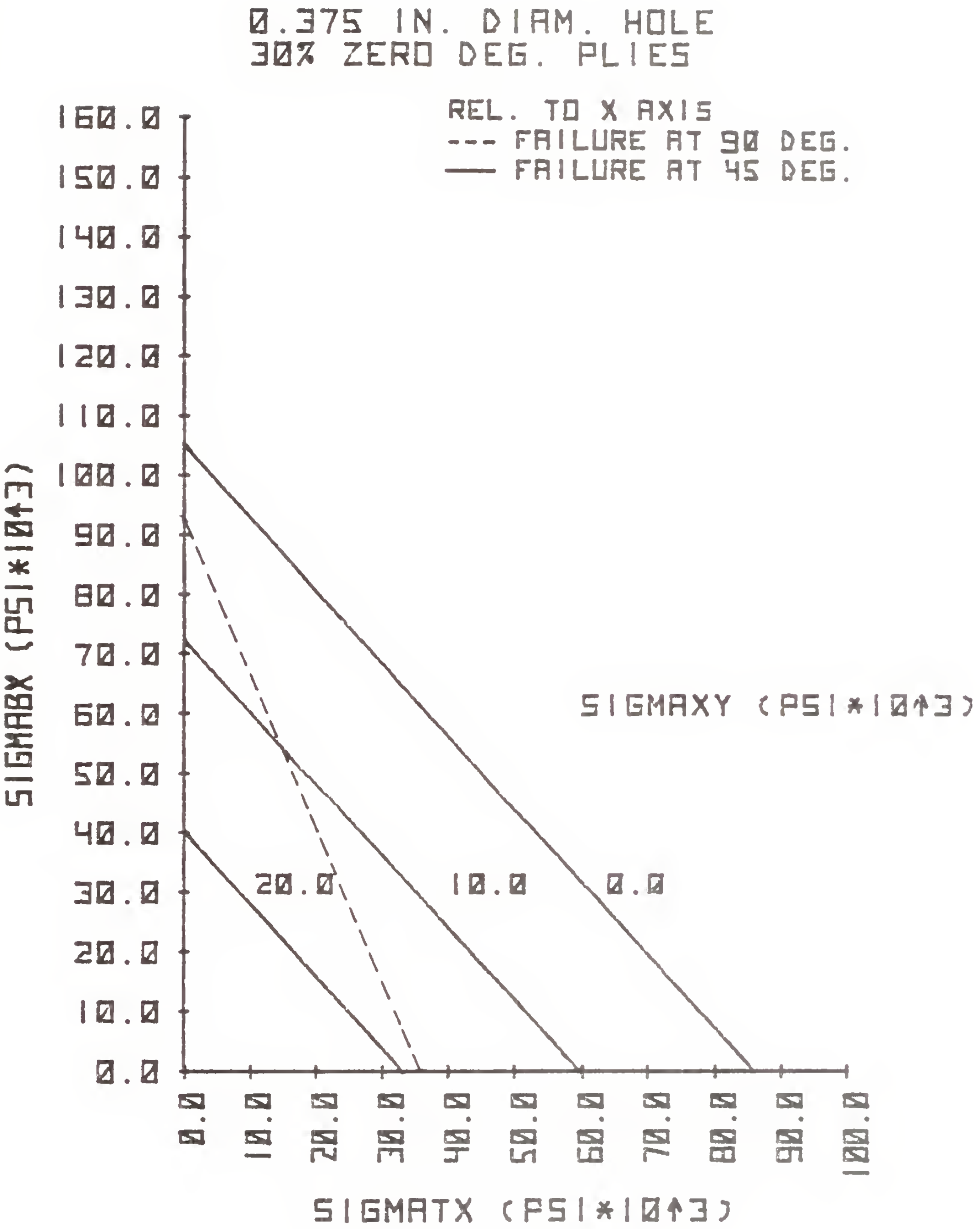


FIGURE 14. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

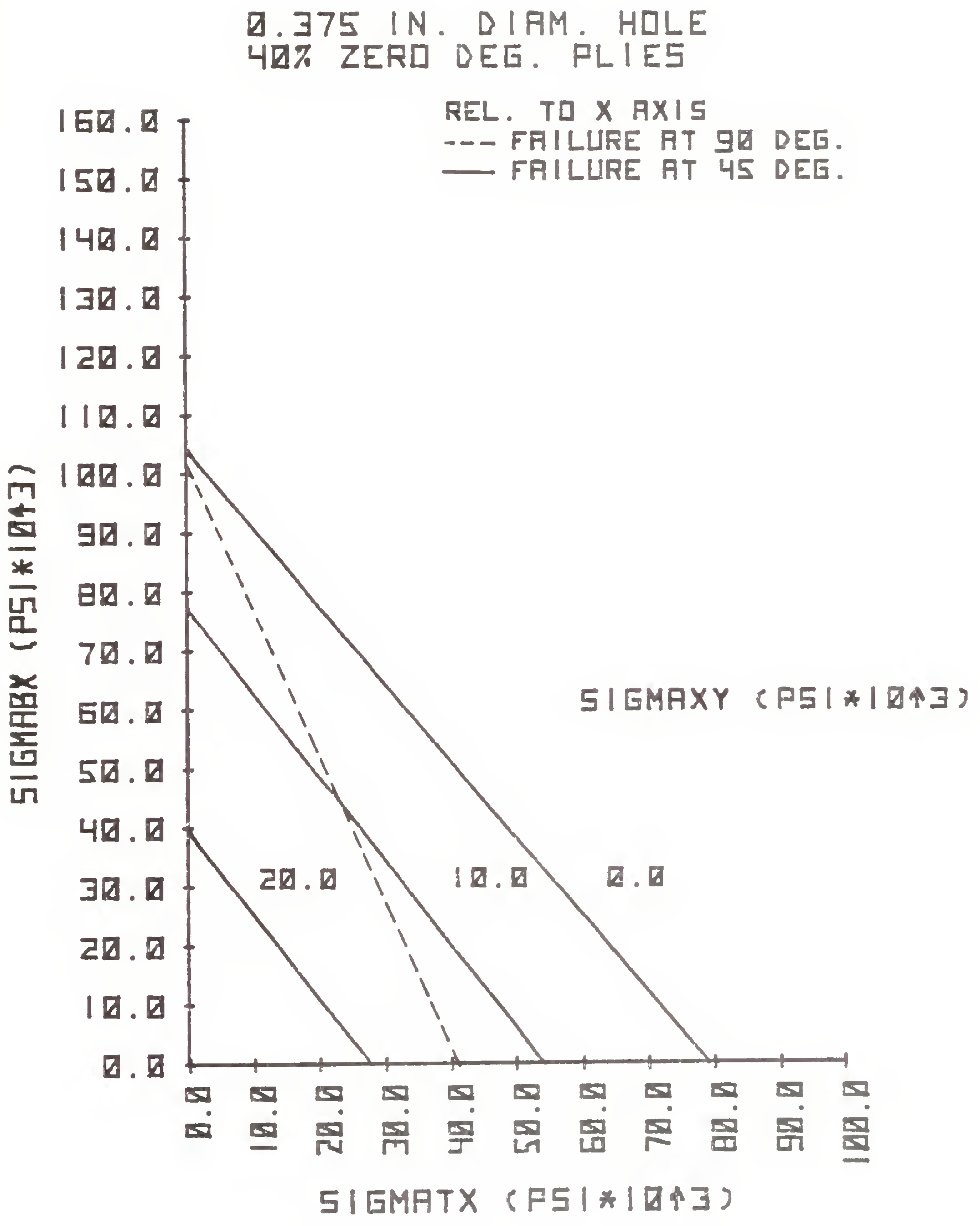


FIGURE 15. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

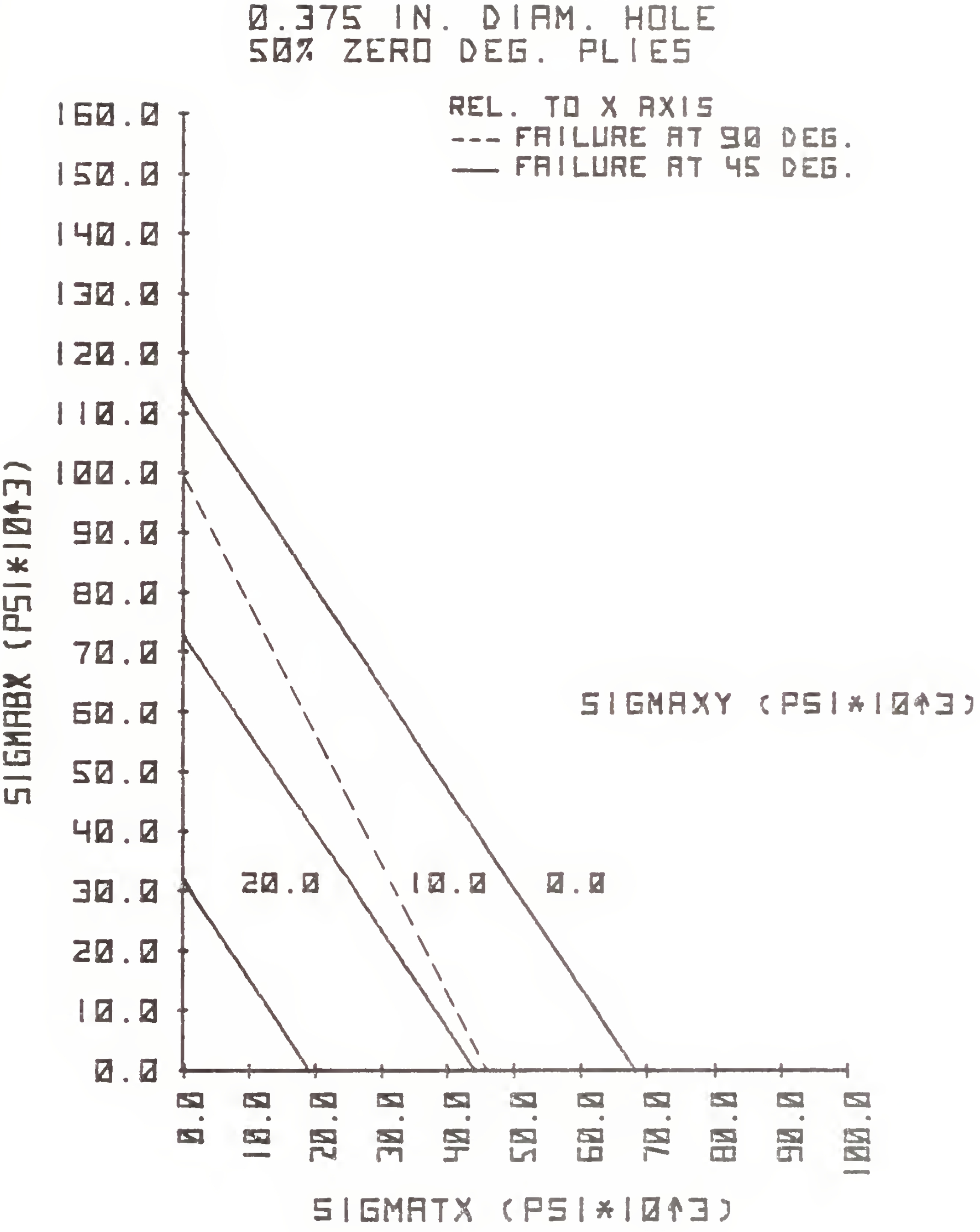


FIGURE 16. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm45]$ MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

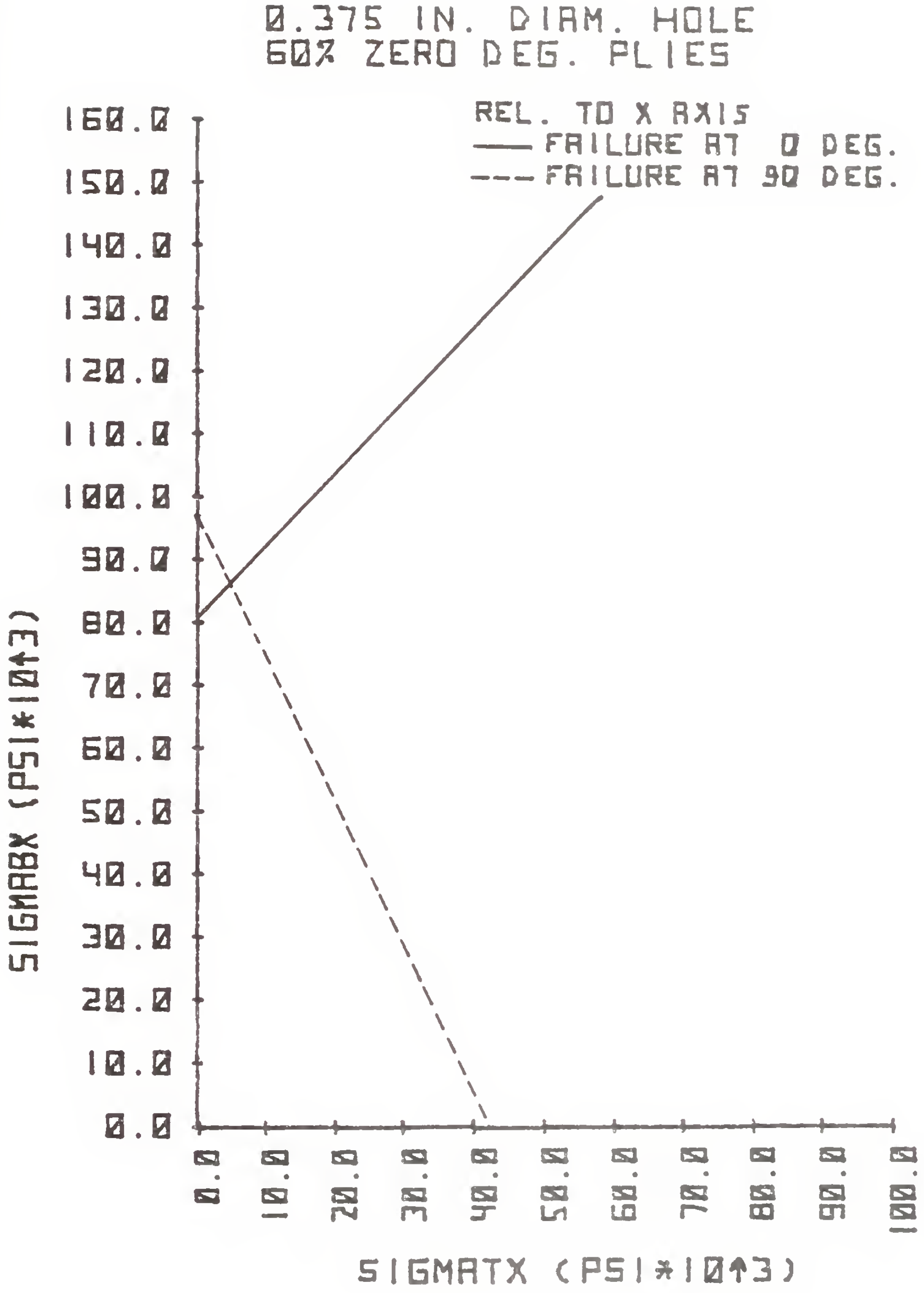


FIGURE 17. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

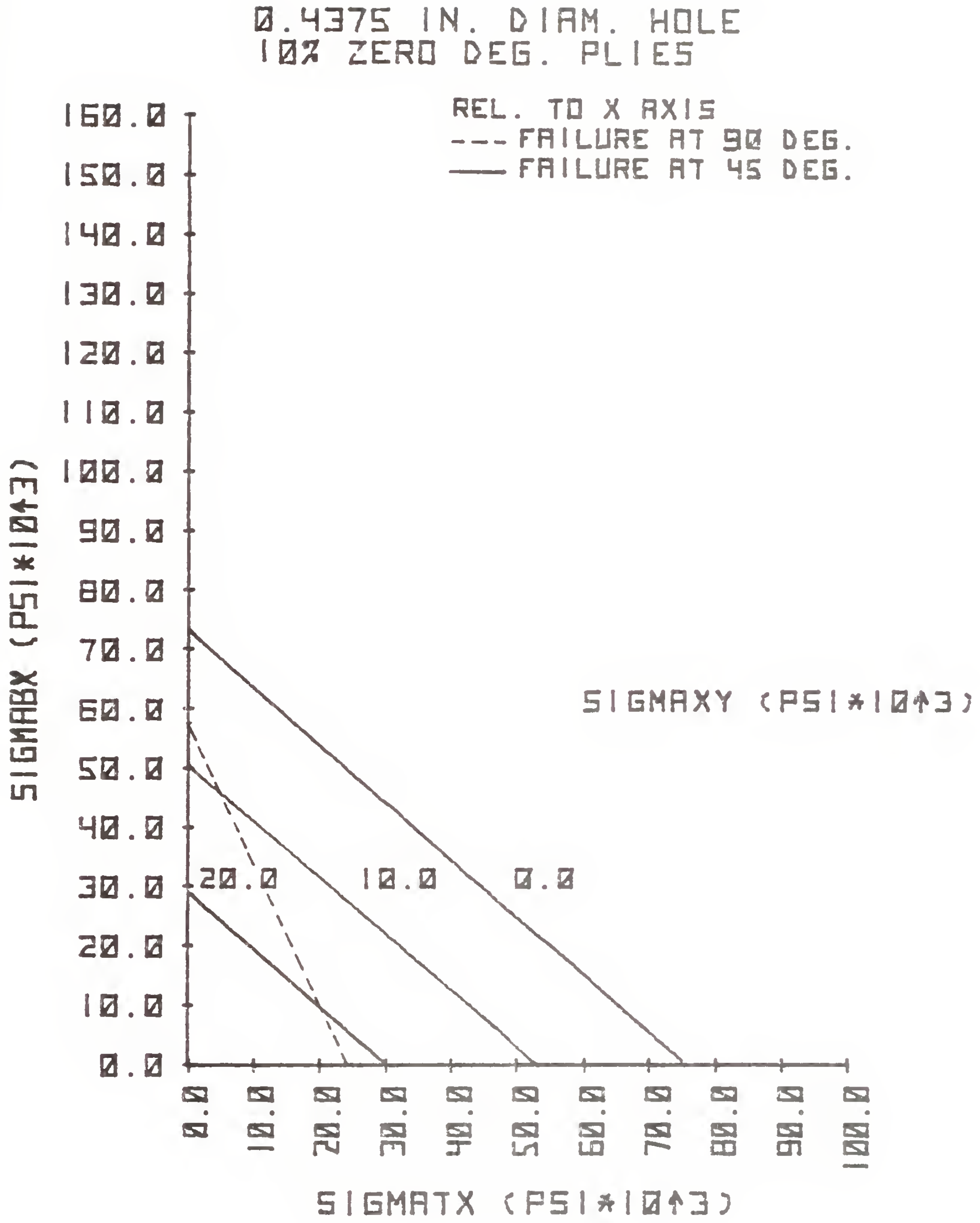


FIGURE 18. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

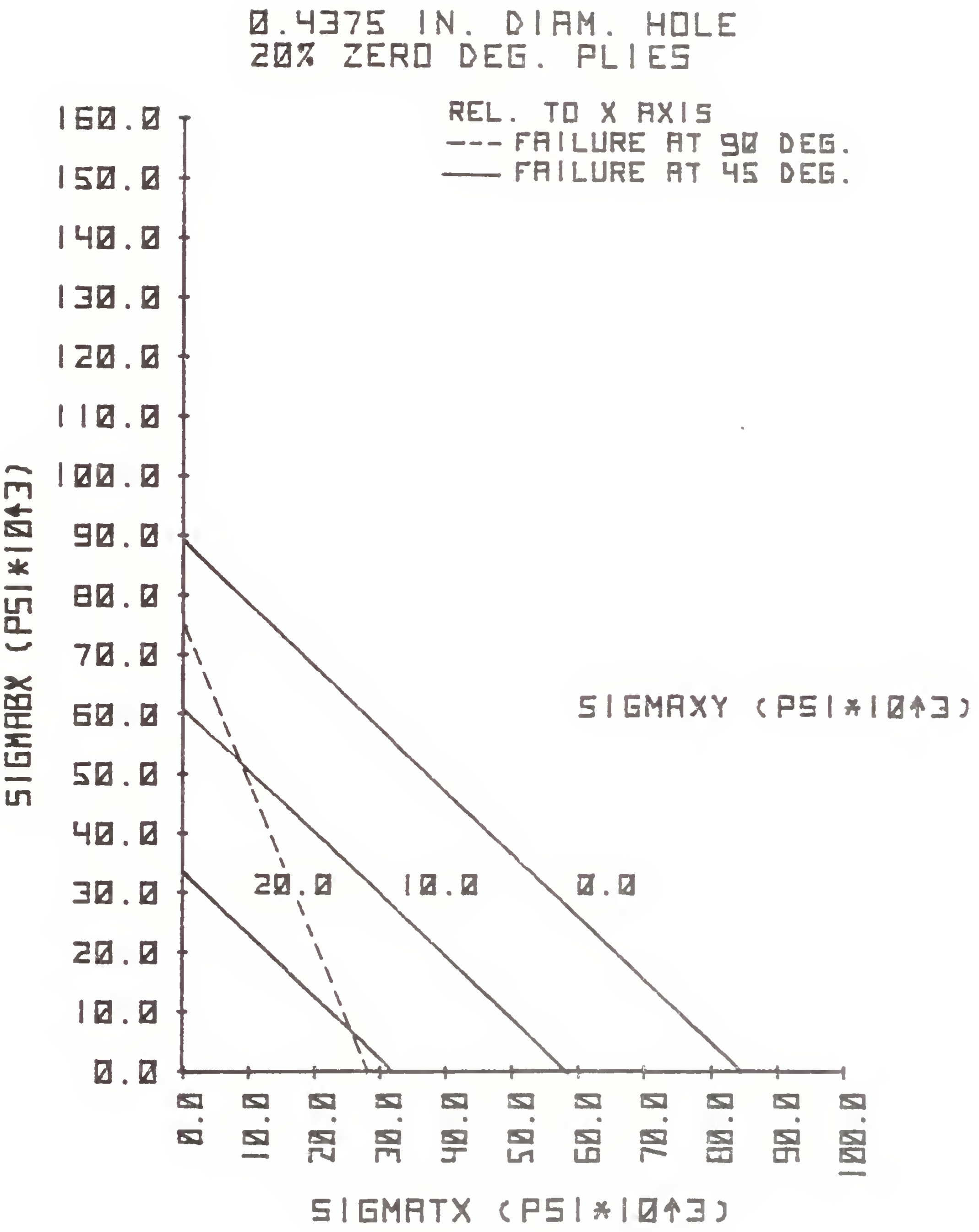


FIGURE 19. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

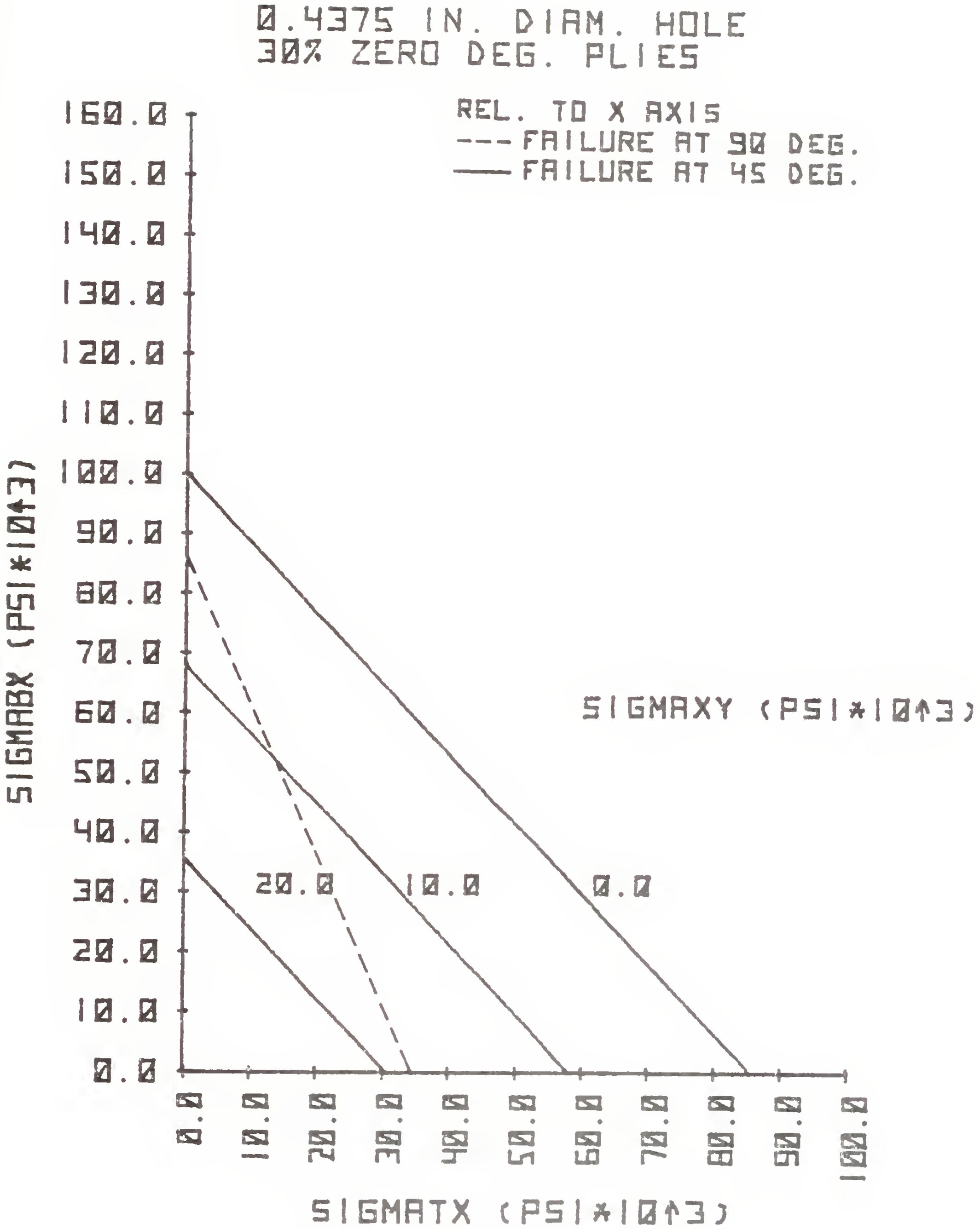


FIGURE 20. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm45]$ MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

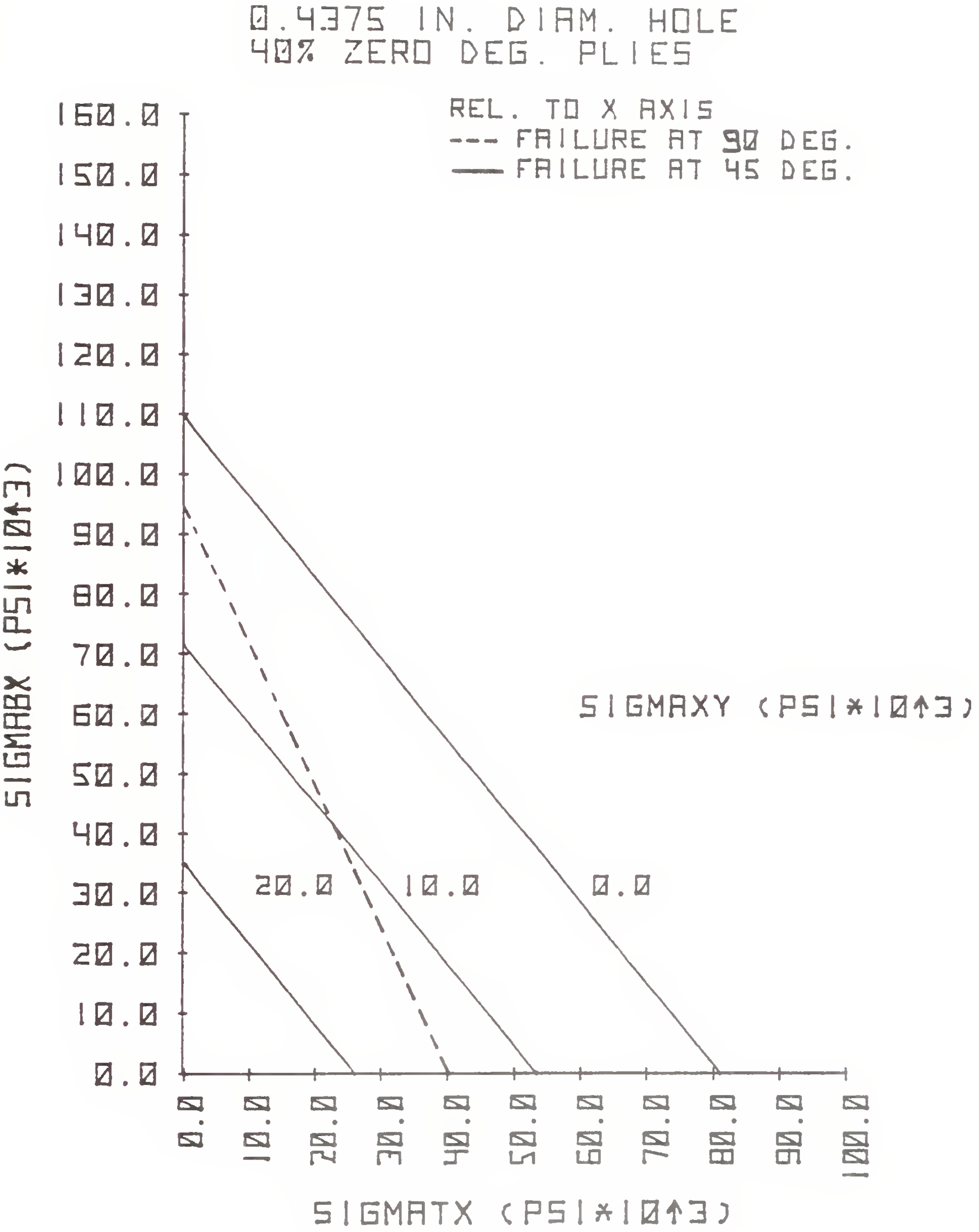


FIGURE 21. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

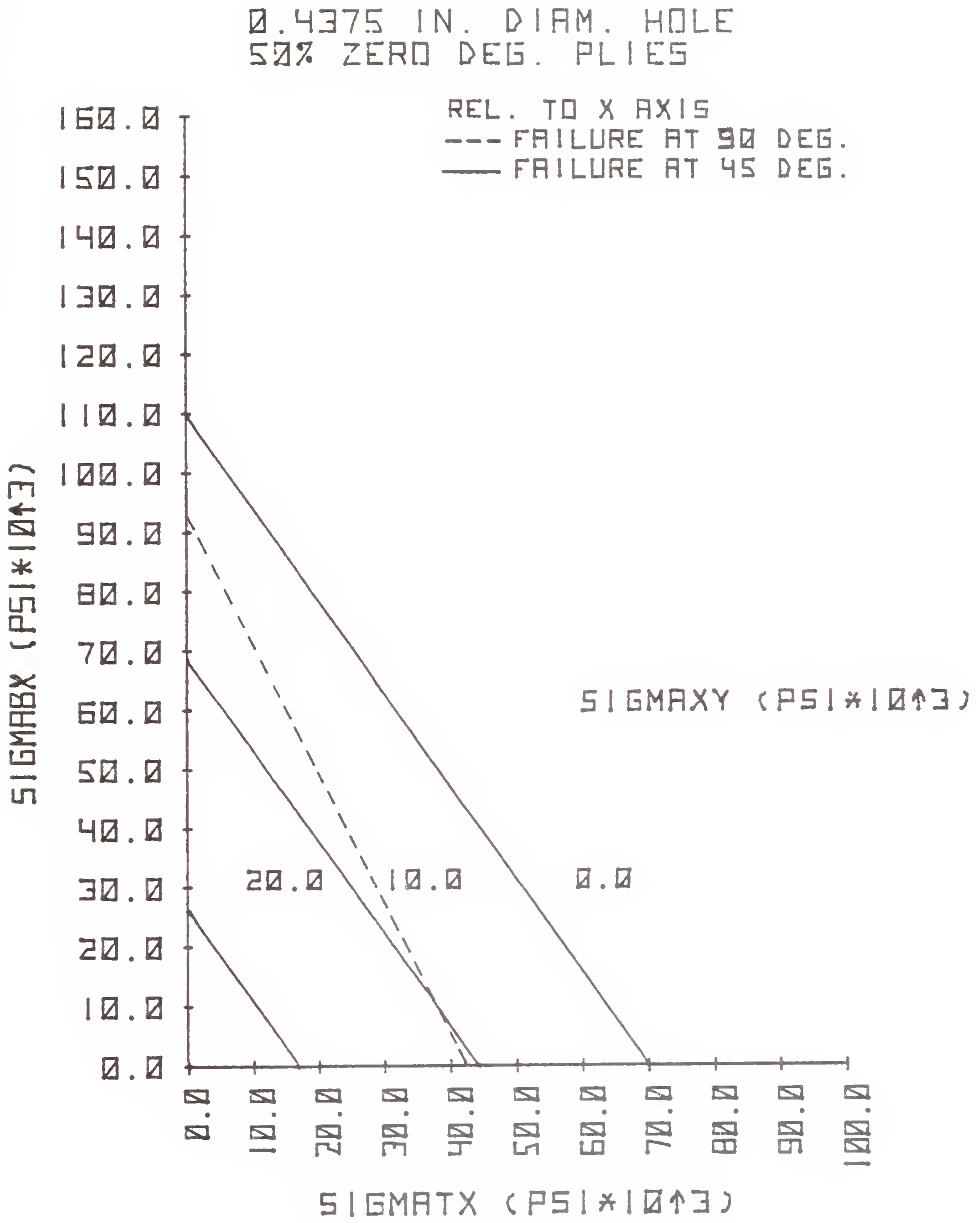


FIGURE 22. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

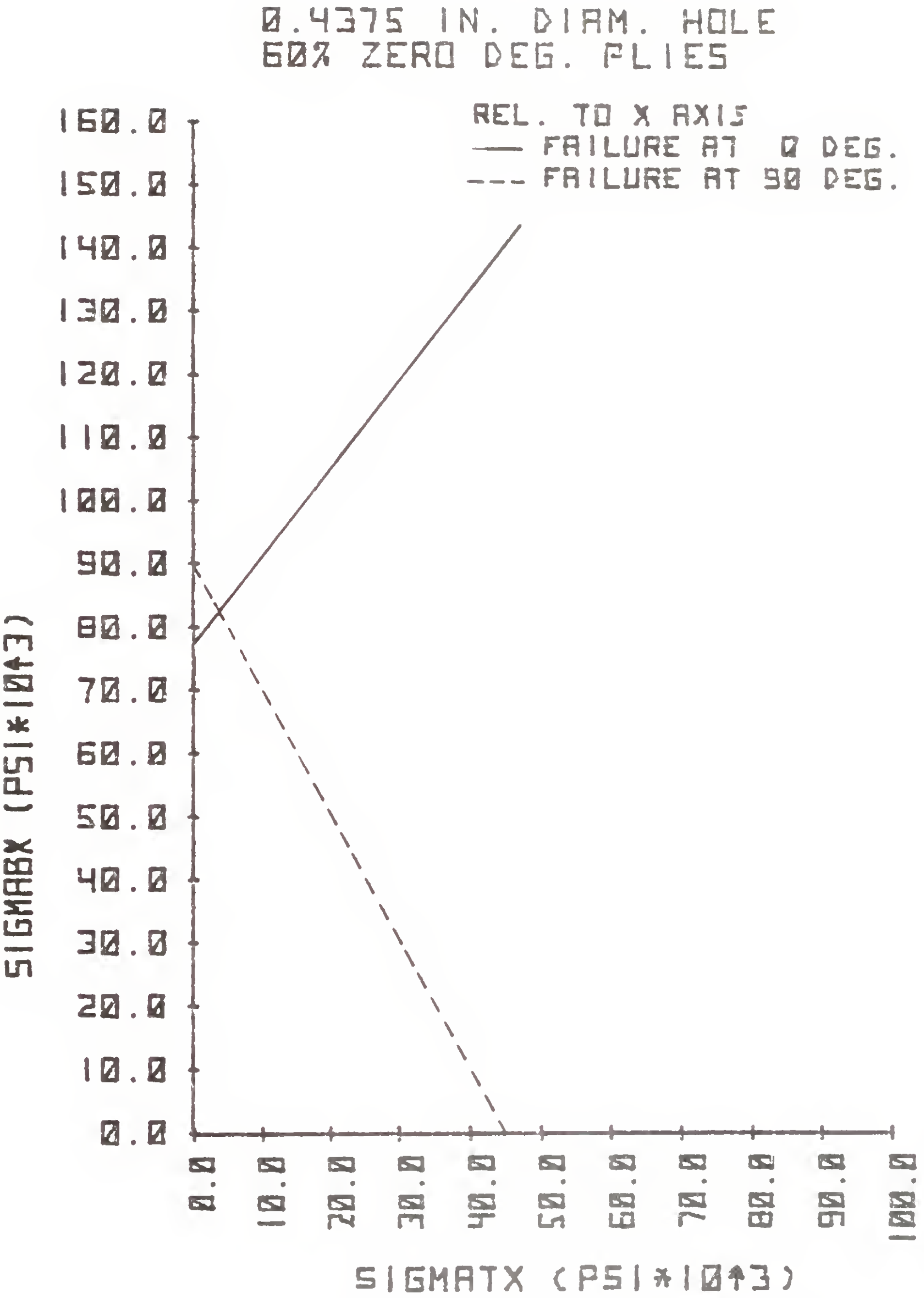


FIGURE 23. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm45]$ MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

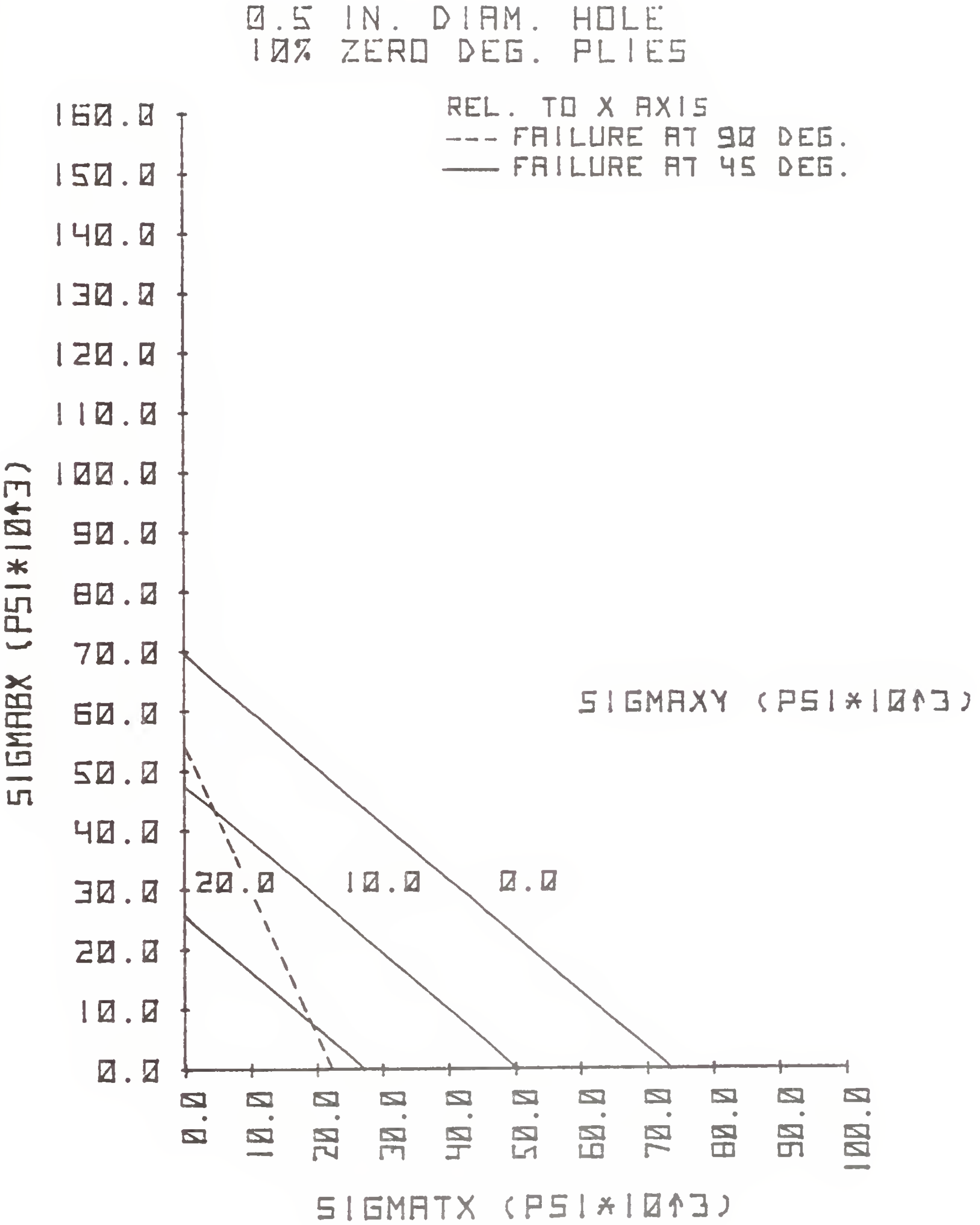


FIGURE 24. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

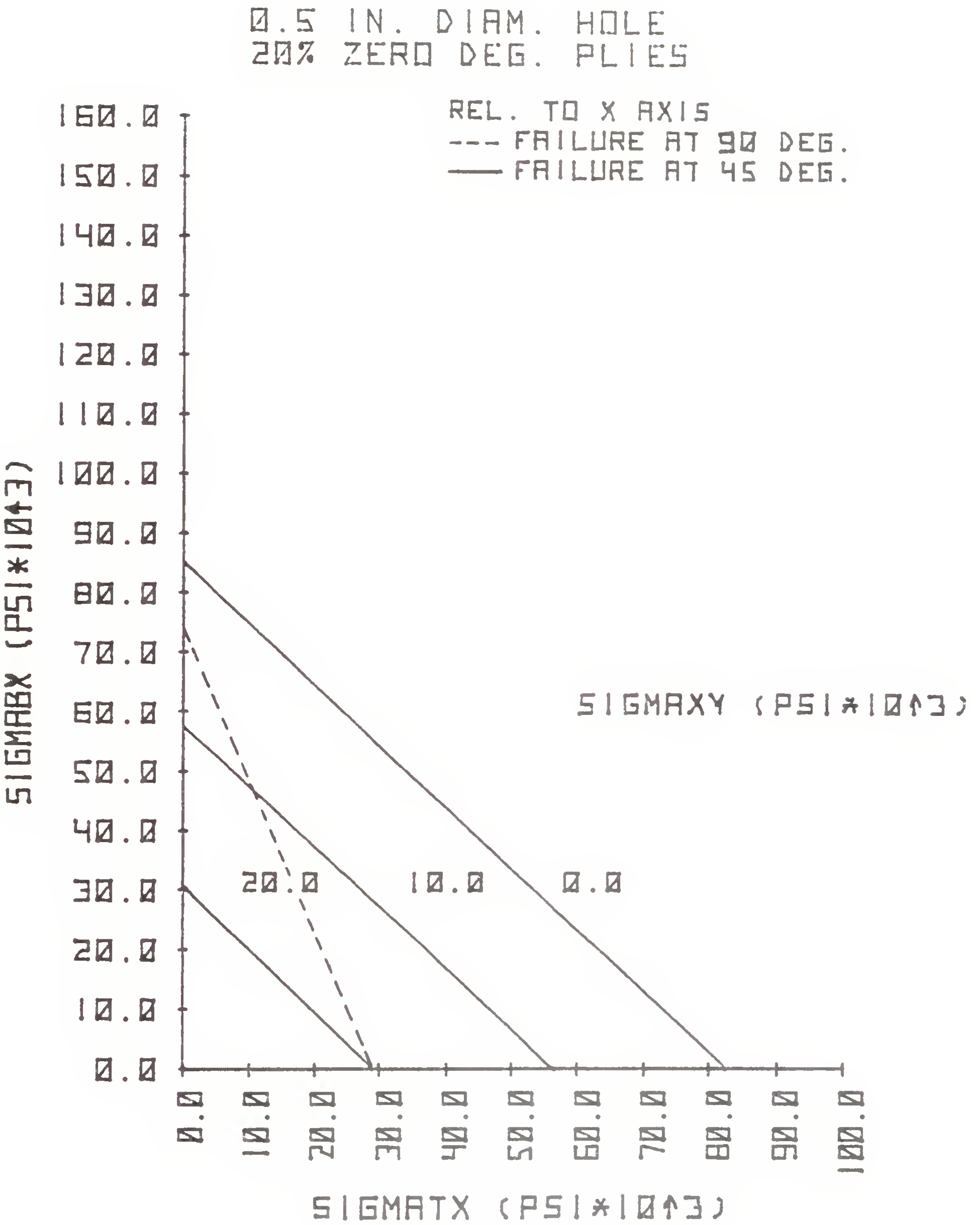


FIGURE 25. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

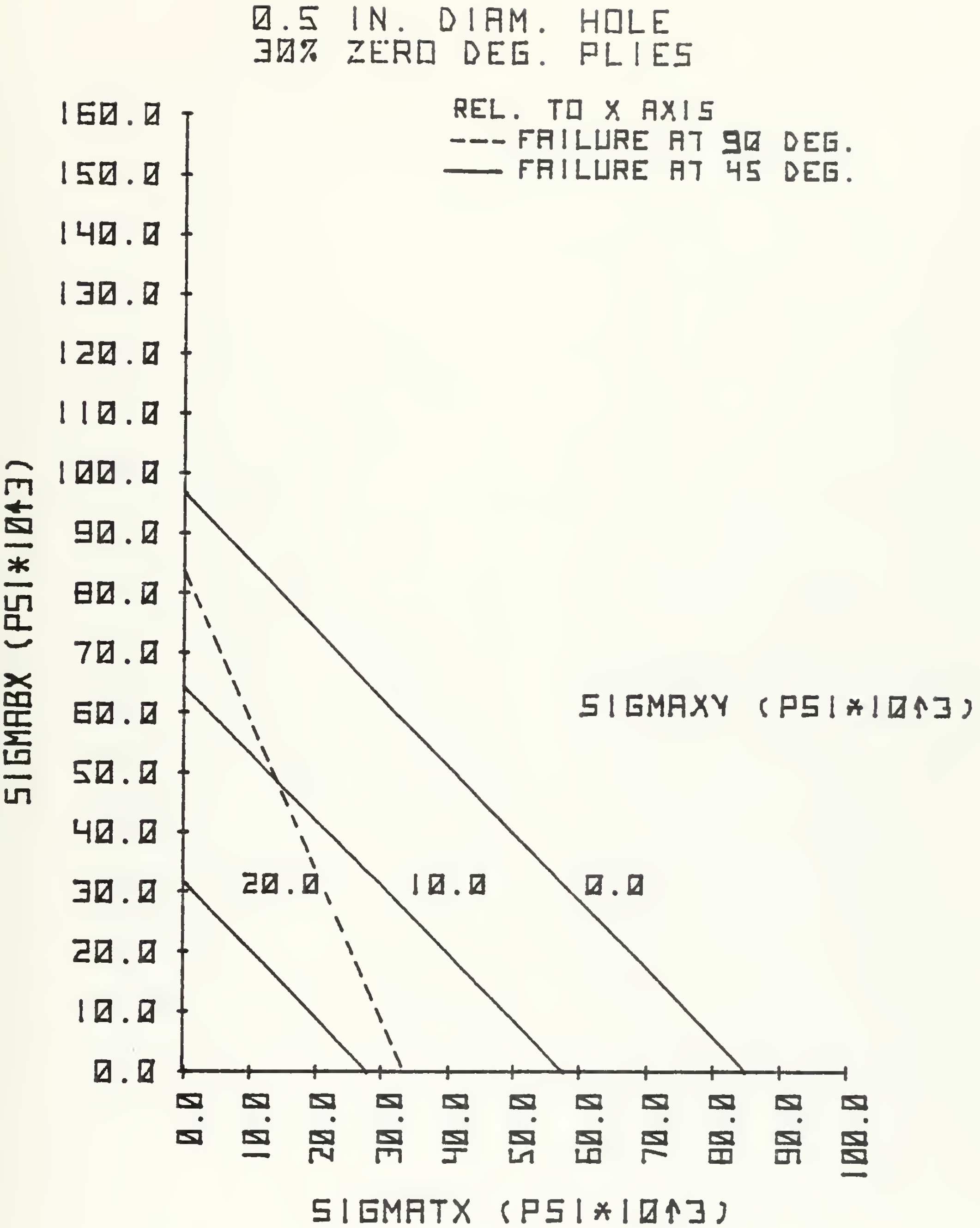


FIGURE 26. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

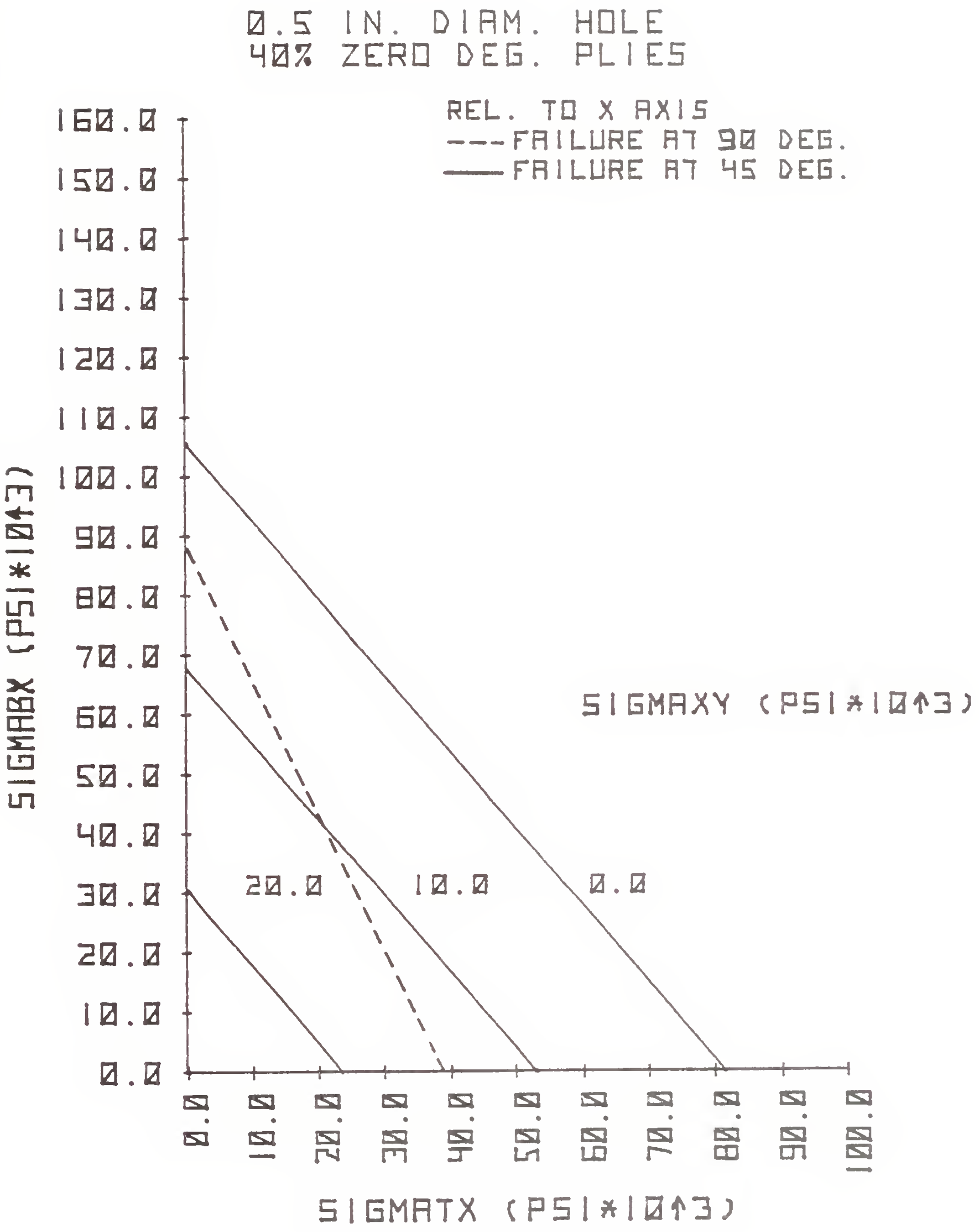


FIGURE 27. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm45]$ MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

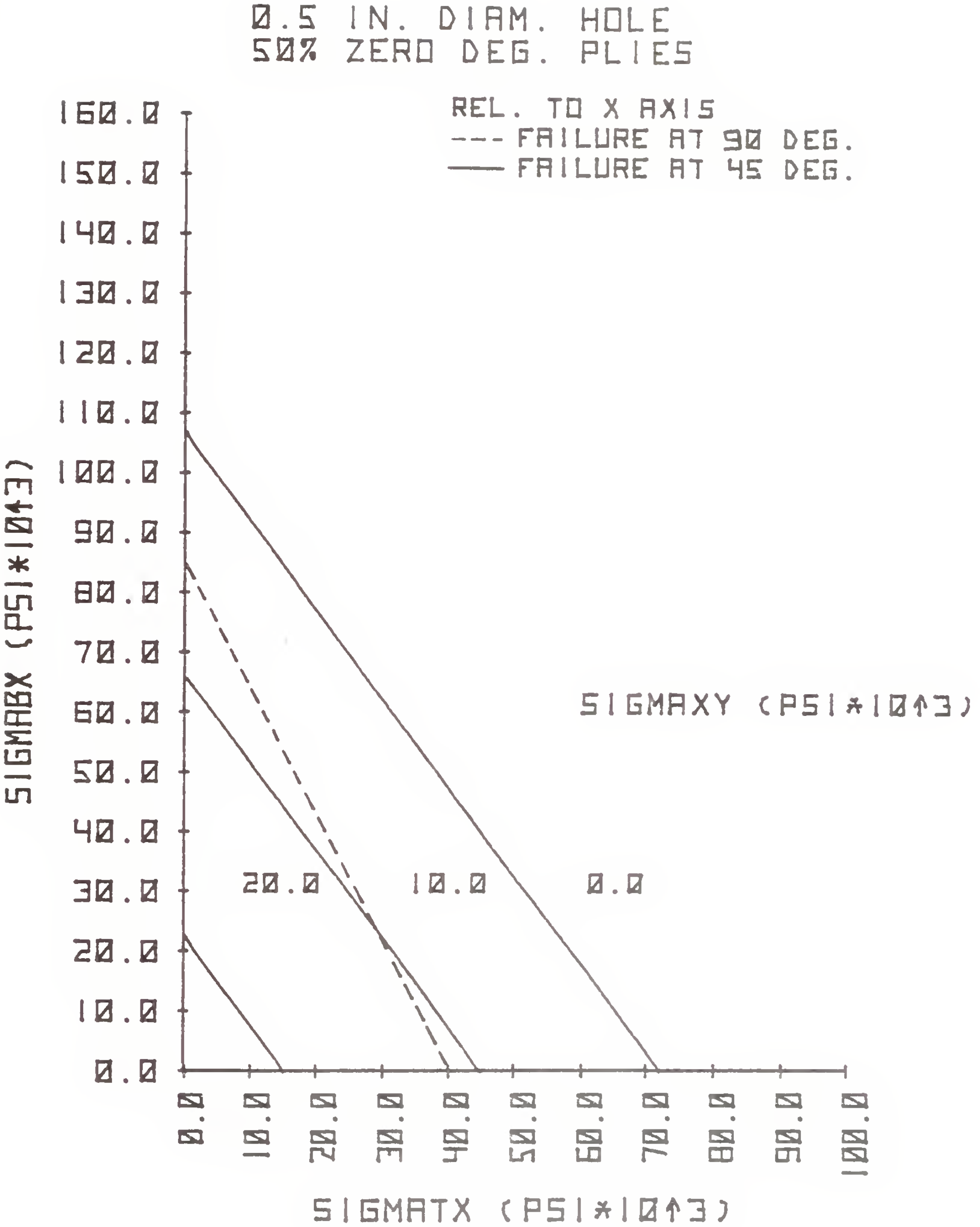


FIGURE 28. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

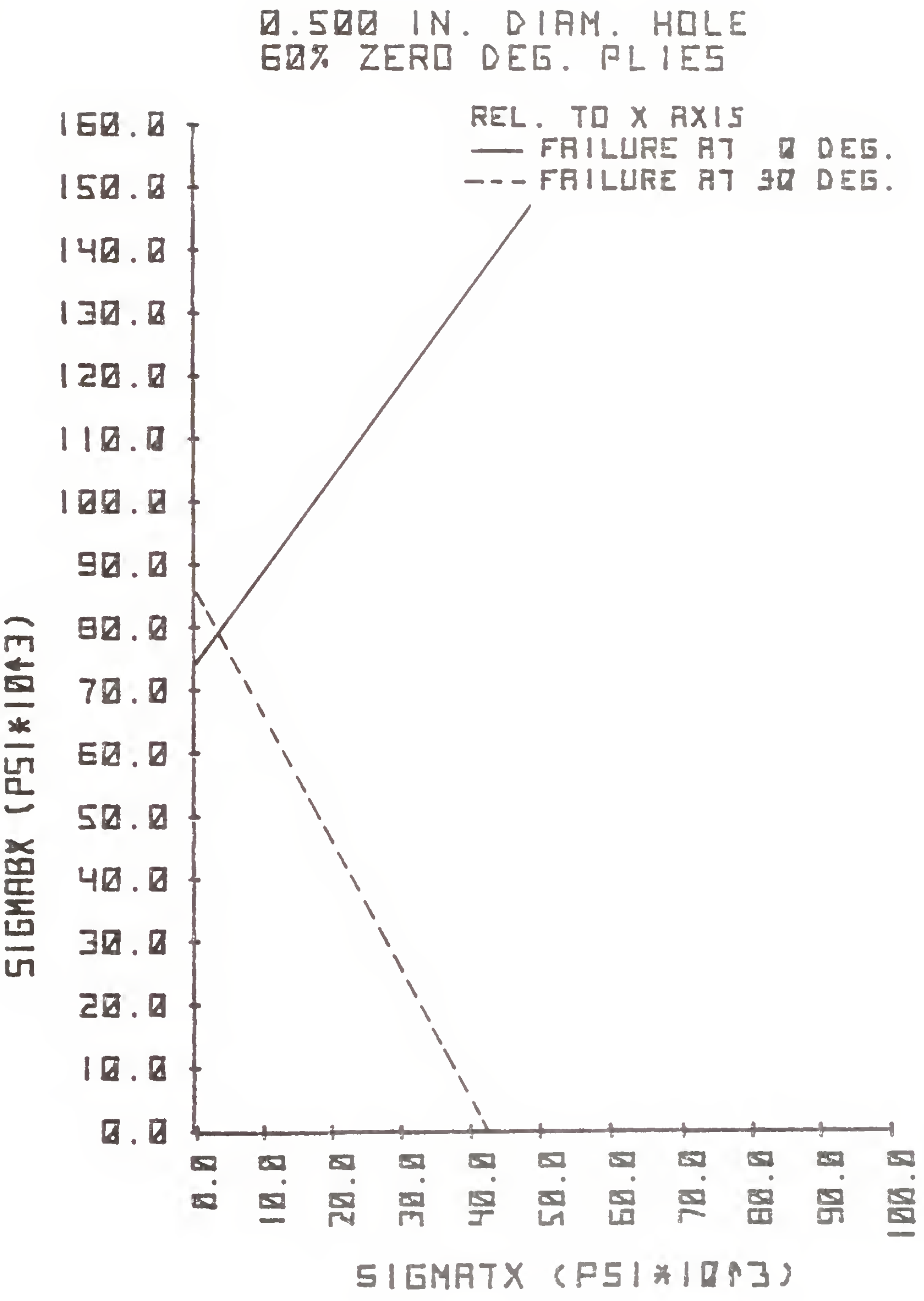


FIGURE 29. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

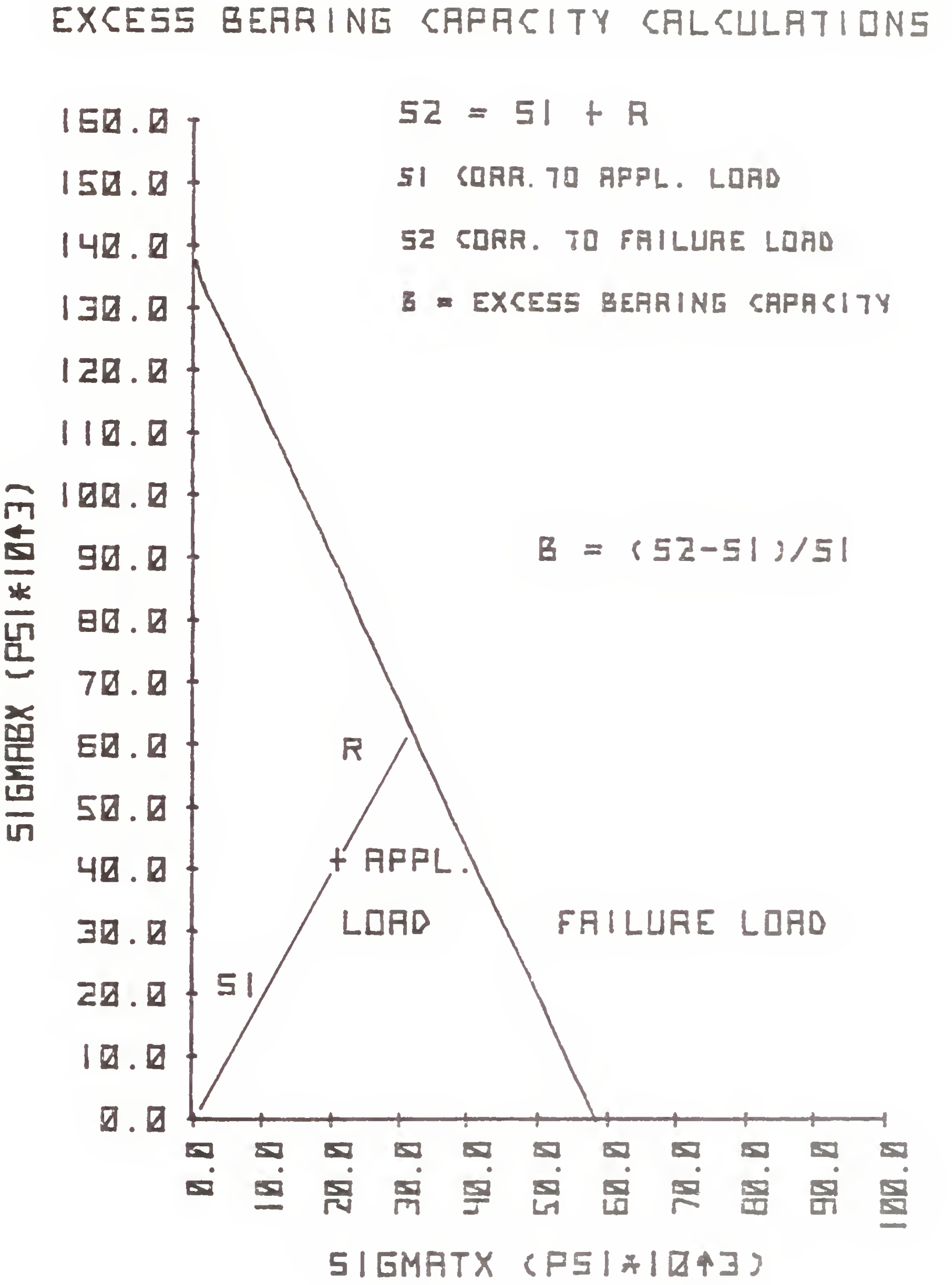


FIGURE 30. EXCESS BEARING CAPACITY CALCULATIONS

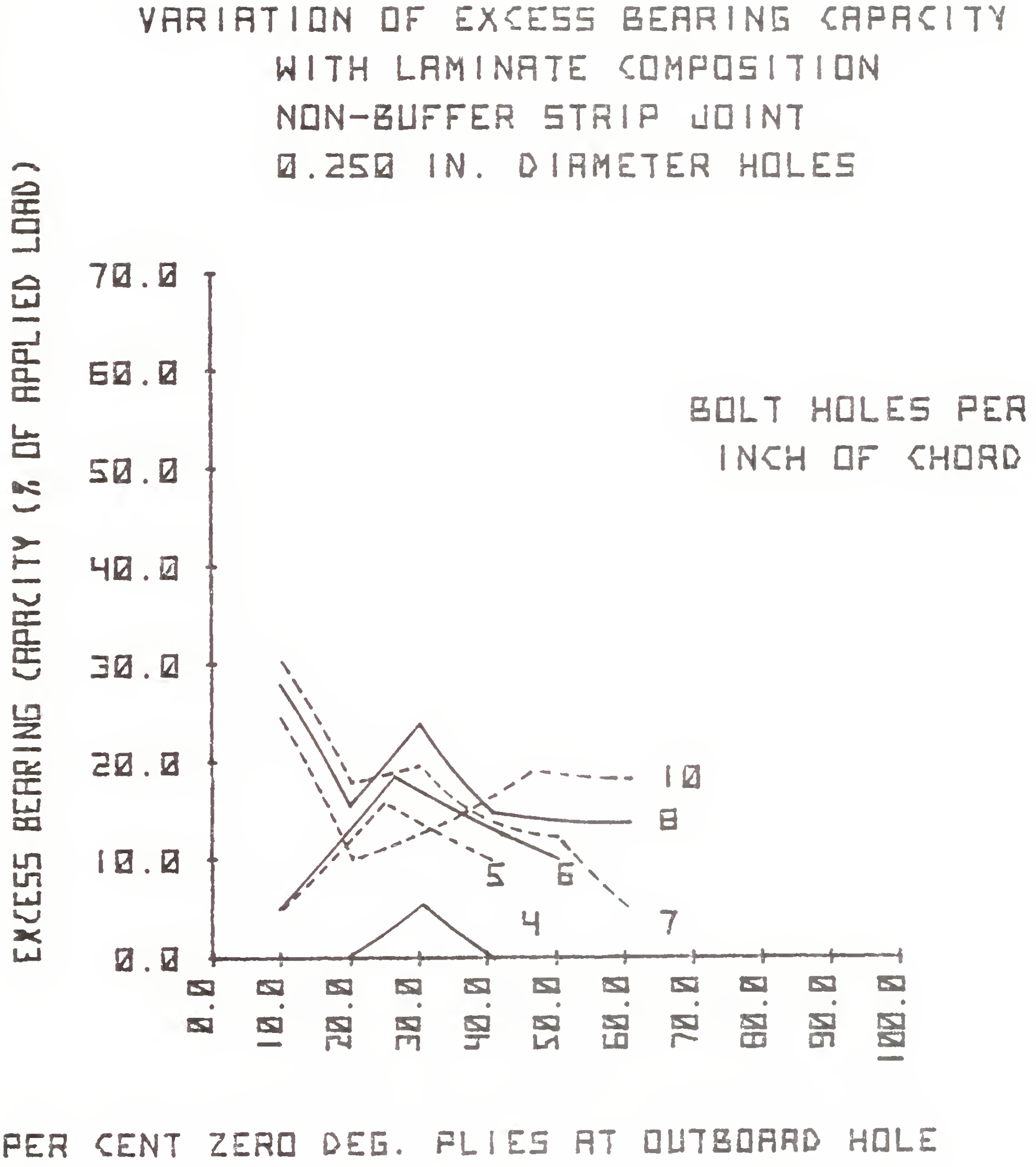


FIGURE 31. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER BOLT HOLES

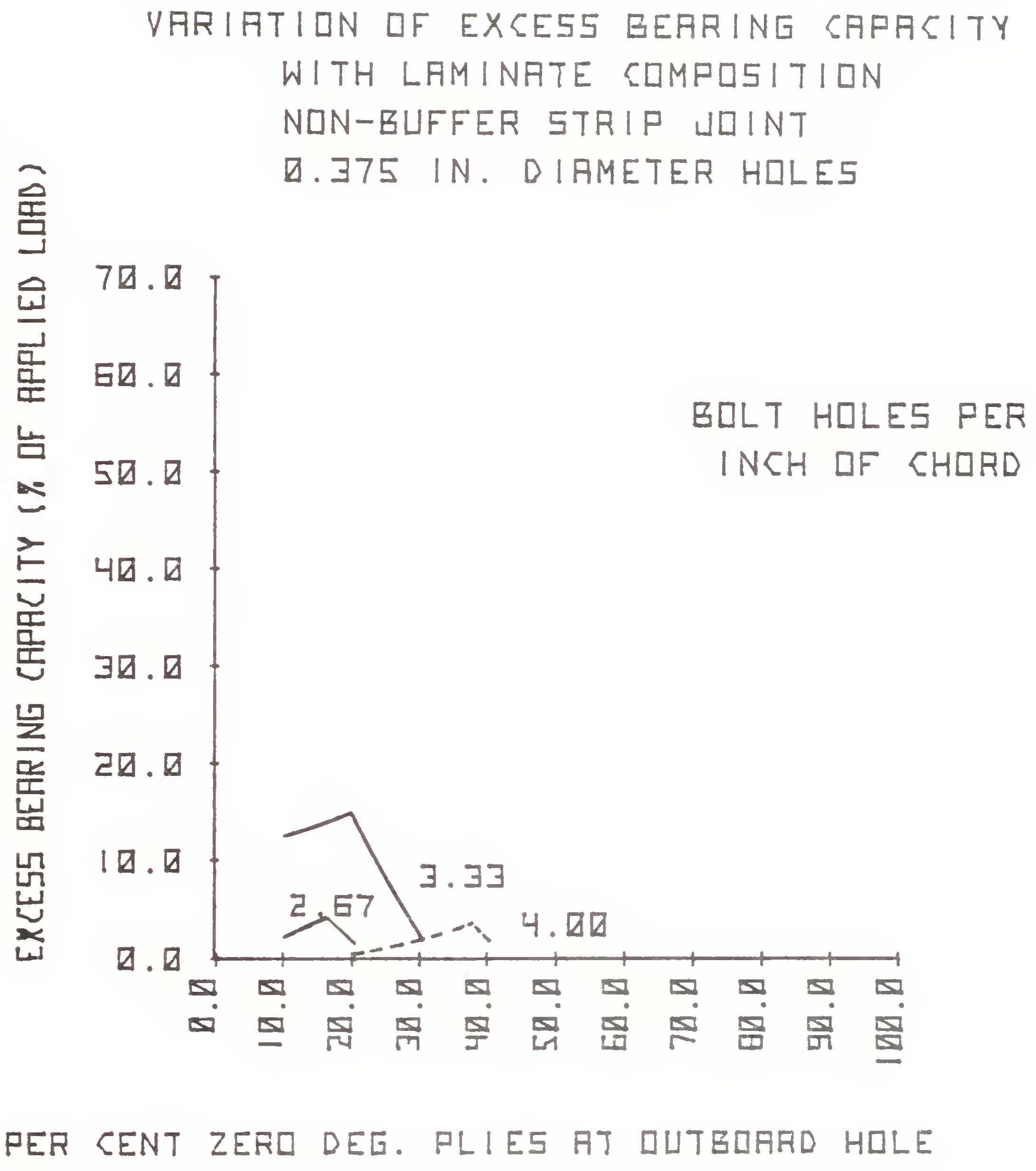


FIGURE 32. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.375 IN. DIAMETER BOLT HOLES

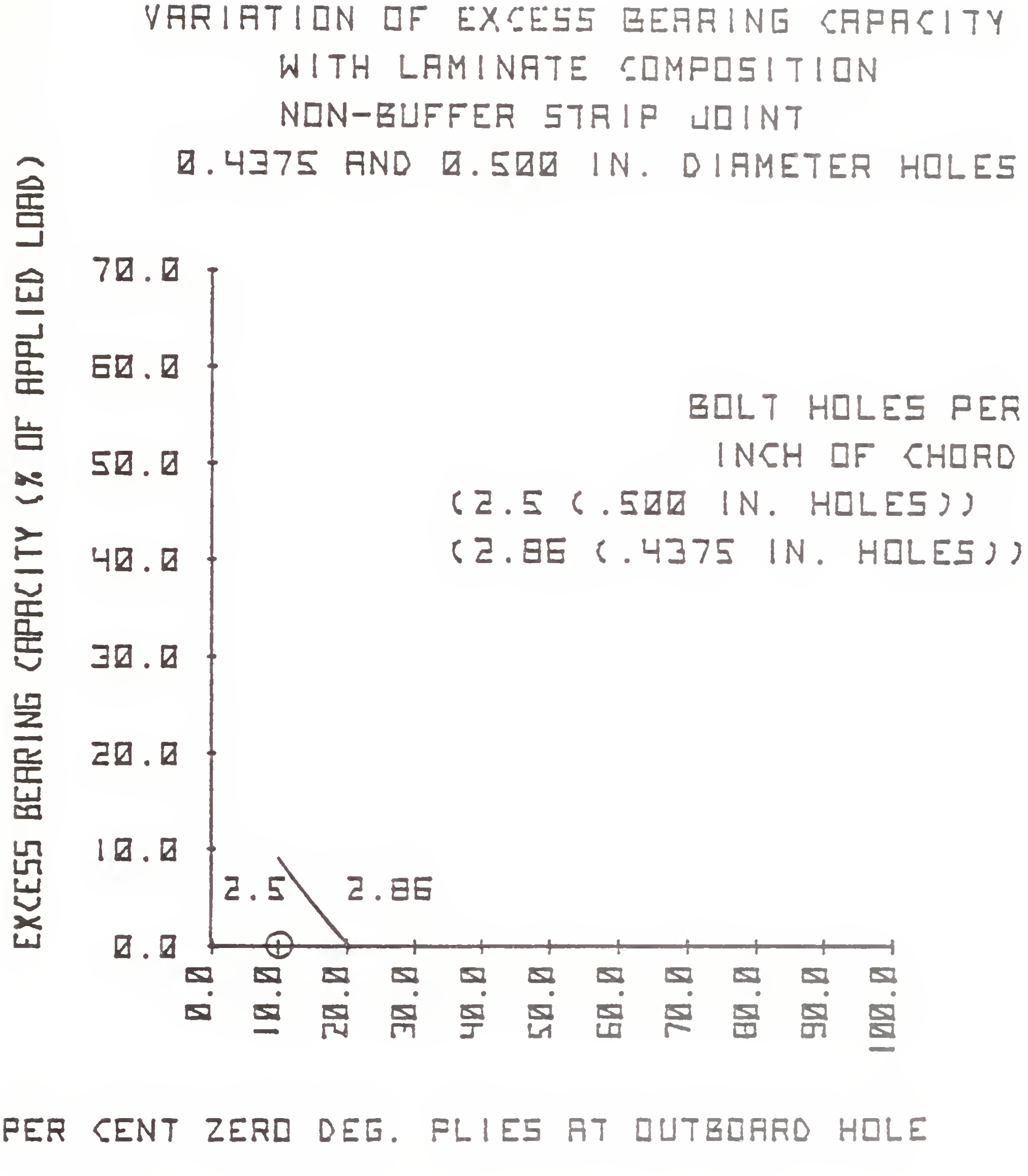


FIGURE 33. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.4375 AND 0.5 IN. DIAMETER BOLT HOLES

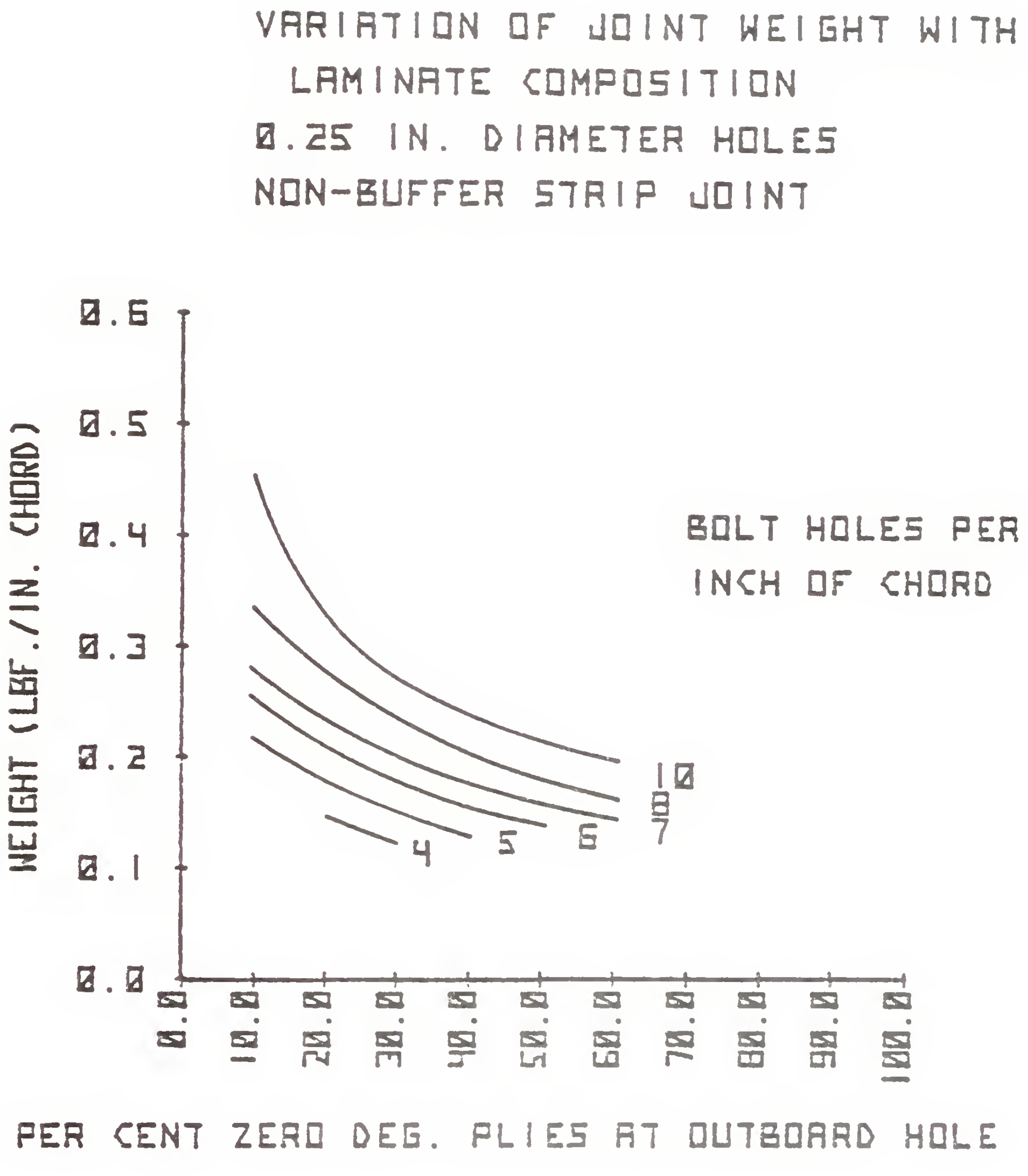


FIGURE 34. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER BOLT HOLES

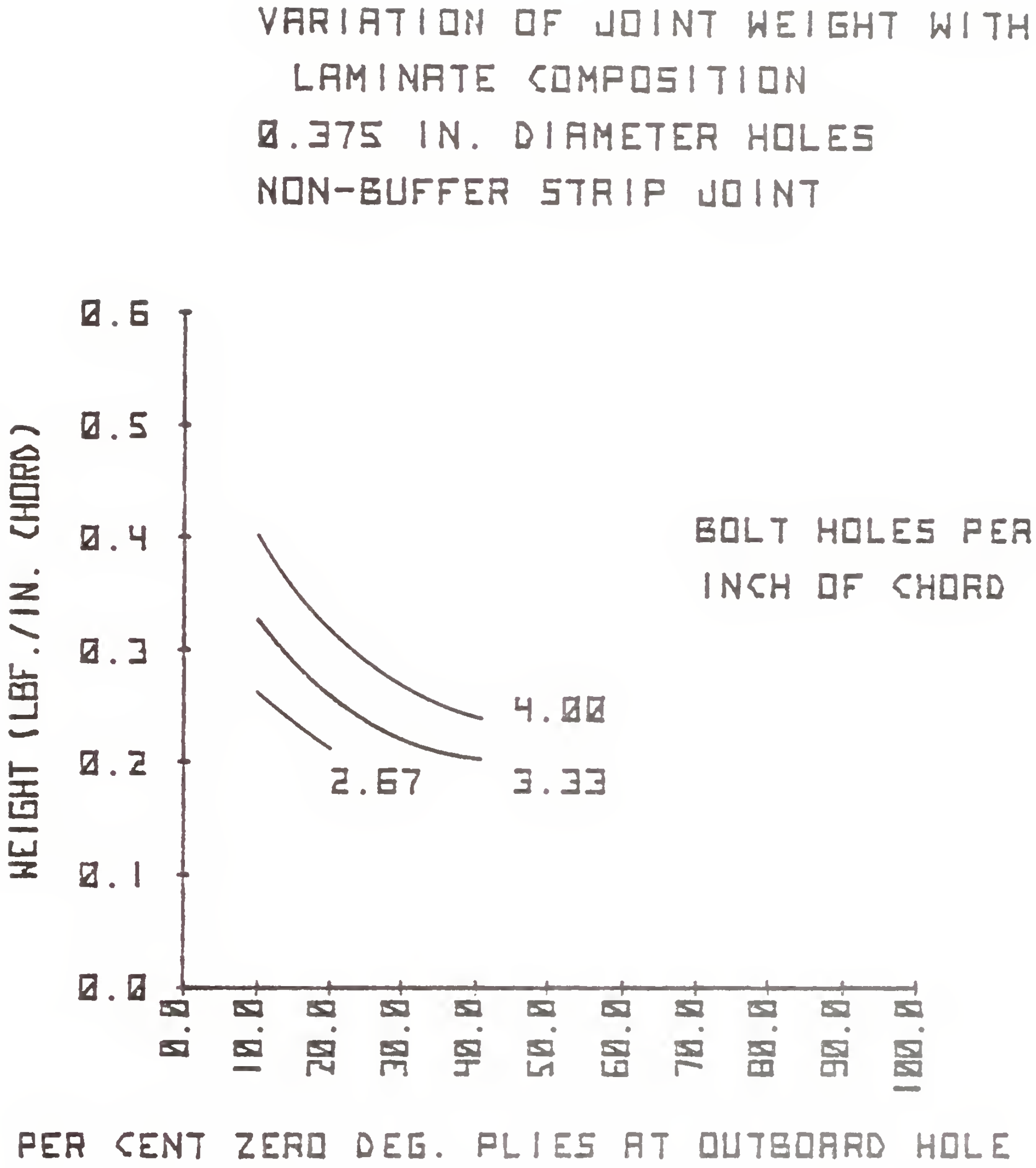


FIGURE 35. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.375 IN. DIAMETER BOLT HOLES

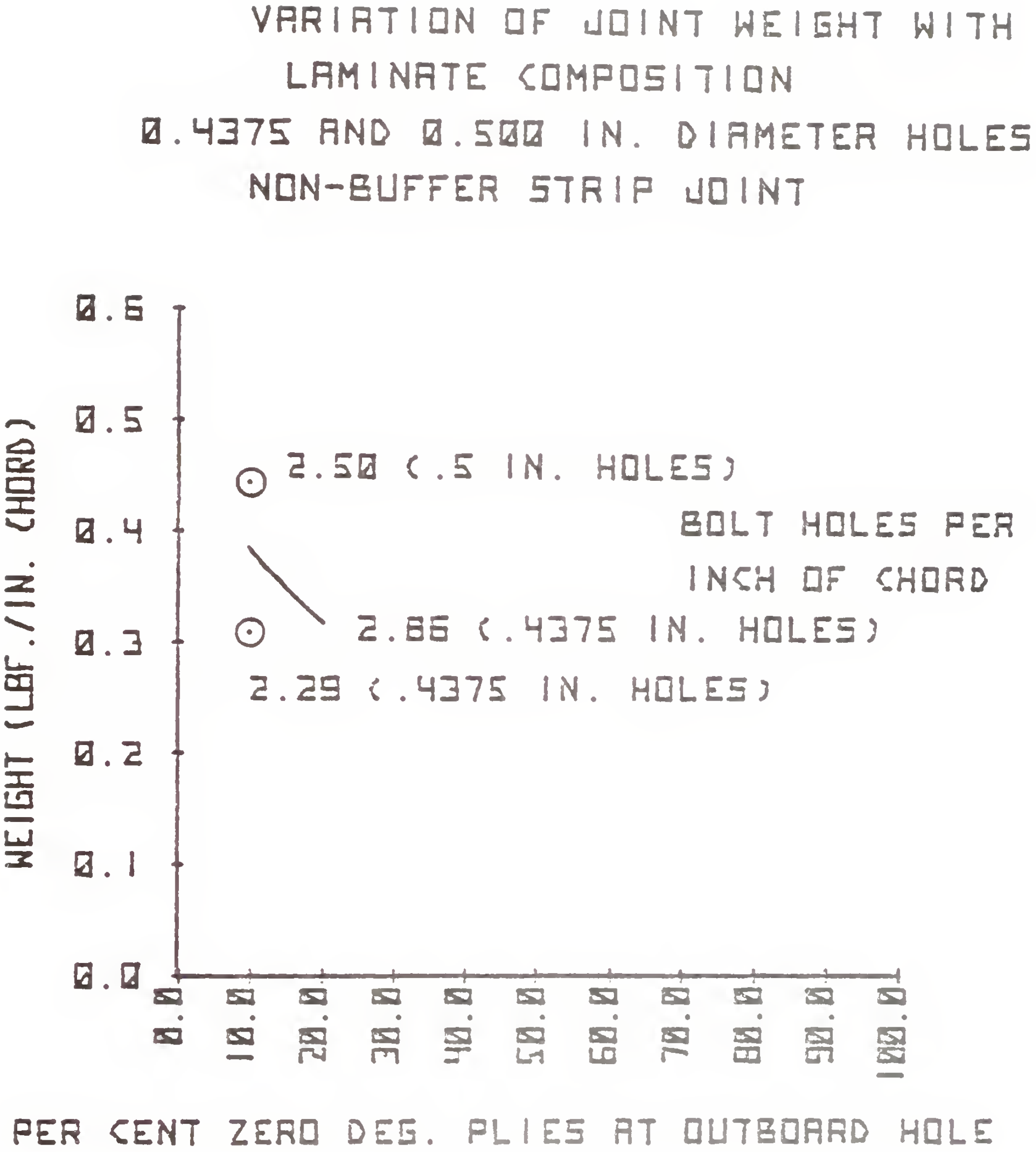
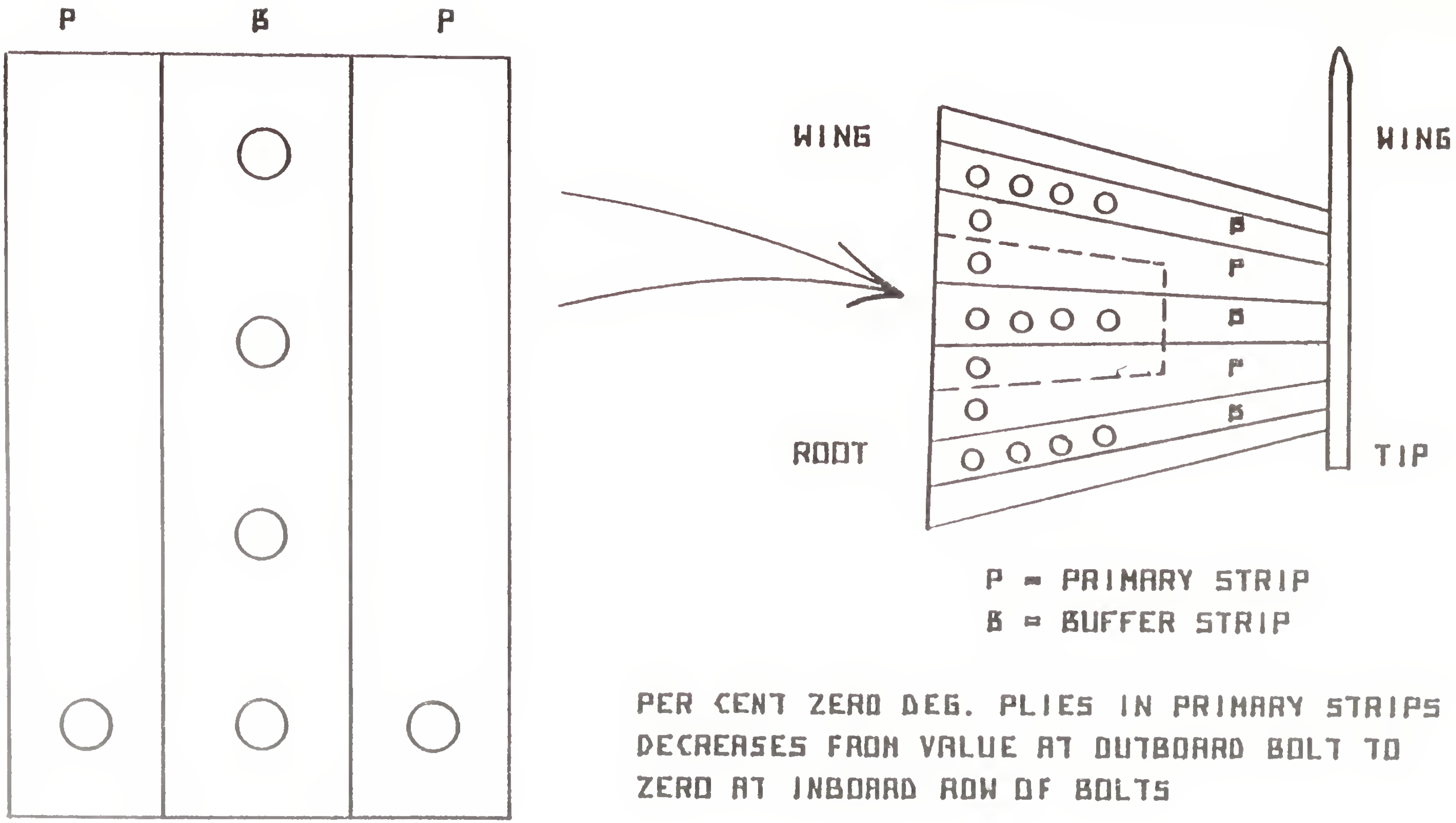


FIGURE 36. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.4375 AND 0.5 IN. DIAMETER BOLT HOLES



P = PRIMARY STRIP
B = BUFFER STRIP

PER CENT ZERO DEG. PLIES IN PRIMARY STRIPS DECREASES FROM VALUE AT OUTBOARD BOLT TO ZERO AT INBOARD ROW OF BOLTS

FIGURE 37. SCHEMATIC OF A WING WITH A BUFFER STRIP JOINT

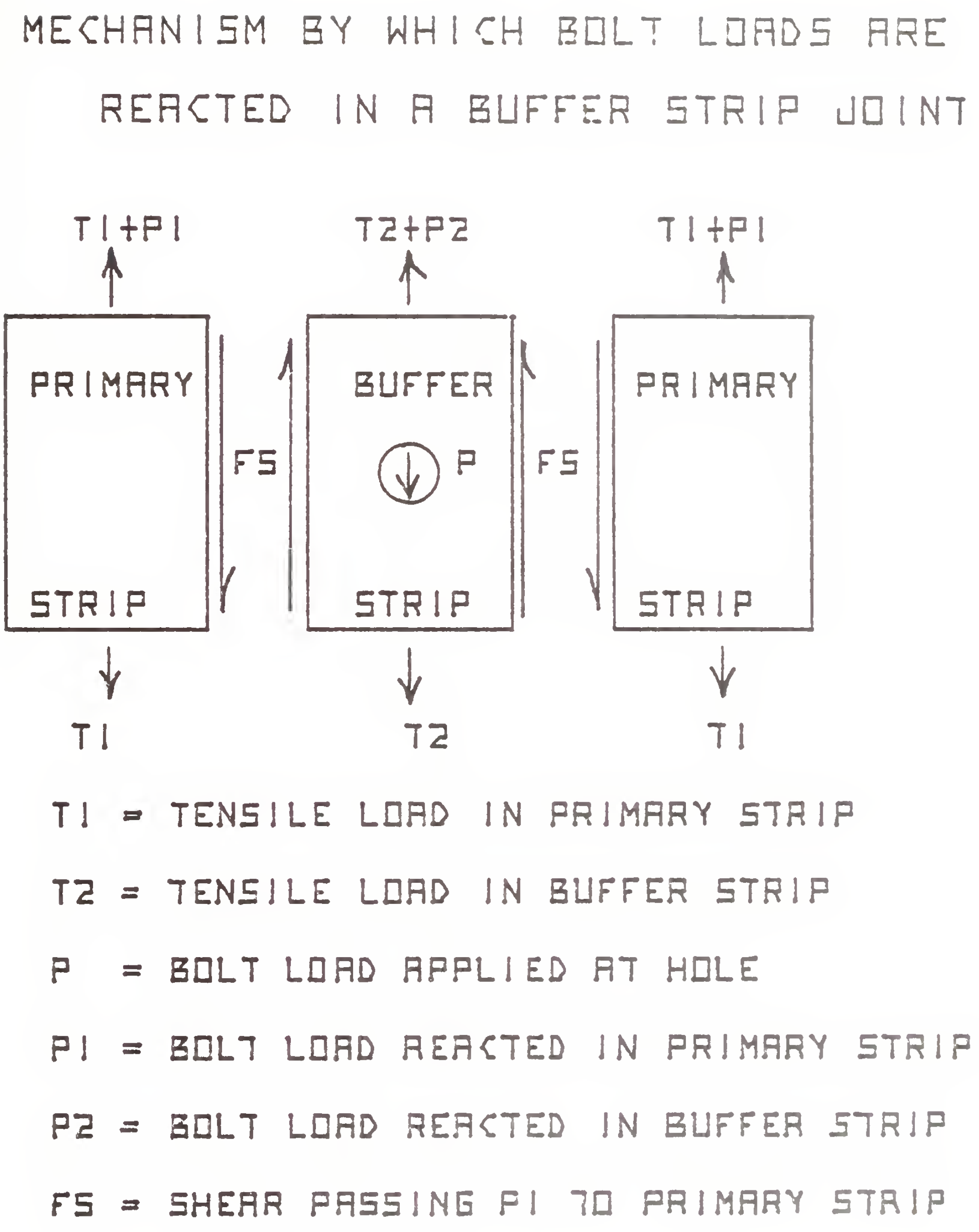

MECHANISM BY WHICH BOLT LOADS ARE
REACTED IN A BUFFER STRIP JOINT
T1+P1
T2+P2
T1+P1
PRIMARY
STRIP
FS
BUFFER
P
STRIP
FS
PRIMARY
STRIP
T1
T2
T1
T1 = TENSILE LOAD IN PRIMARY STRIP
T2 = TENSILE LOAD IN BUFFER STRIP
P = BOLT LOAD APPLIED AT HOLE
P1 = BOLT LOAD REACTED IN PRIMARY STRIP
P2 = BOLT LOAD REACTED IN BUFFER STRIP
FS = SHEAR PASSING P1 TO PRIMARY STRIP

FIGURE 38. MECHANISM BY WHICH BOLT LOADS ARE REACTED IN A BUFFER STRIP JOINT

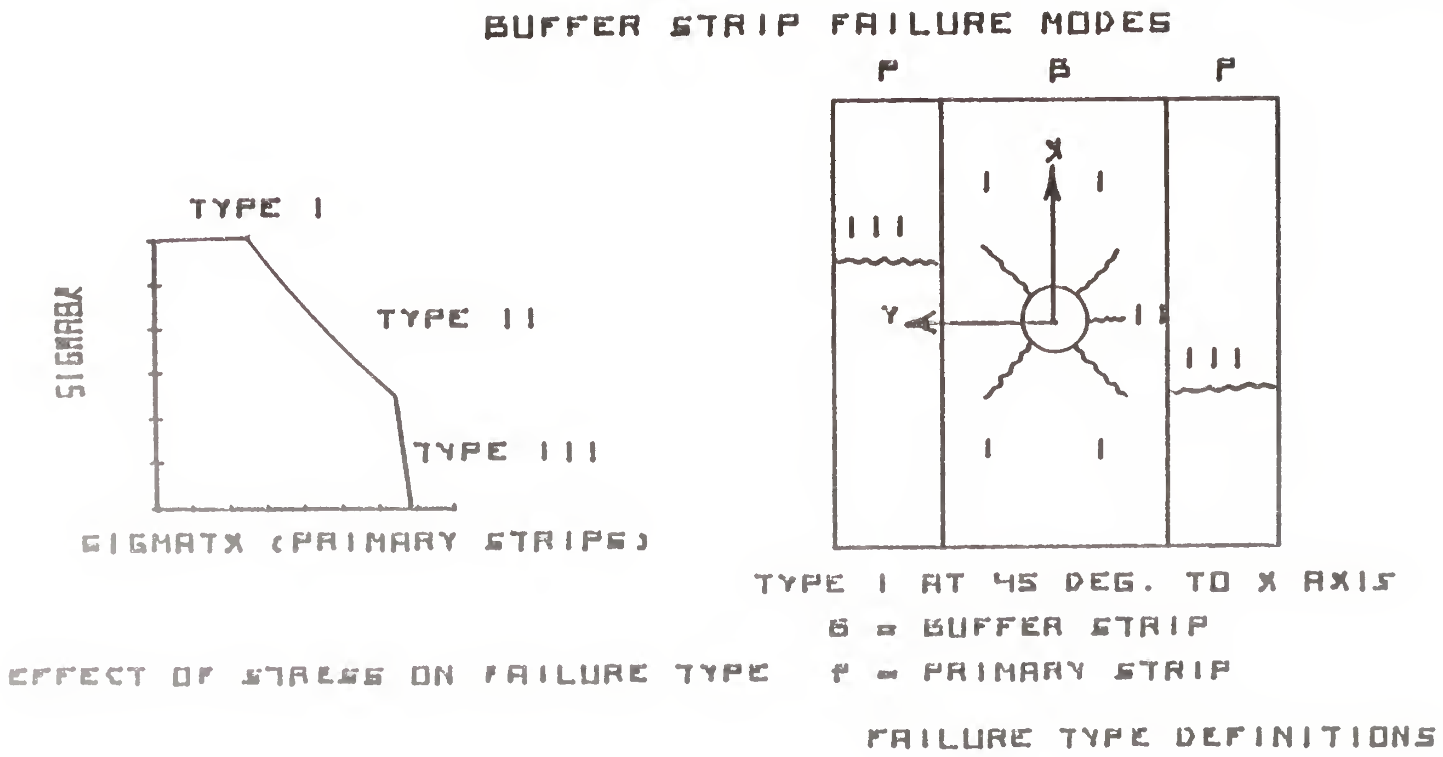


FIGURE 39. DESCRIPTION OF THE EXPECTED BUFFER STRIP FAILURE MODES

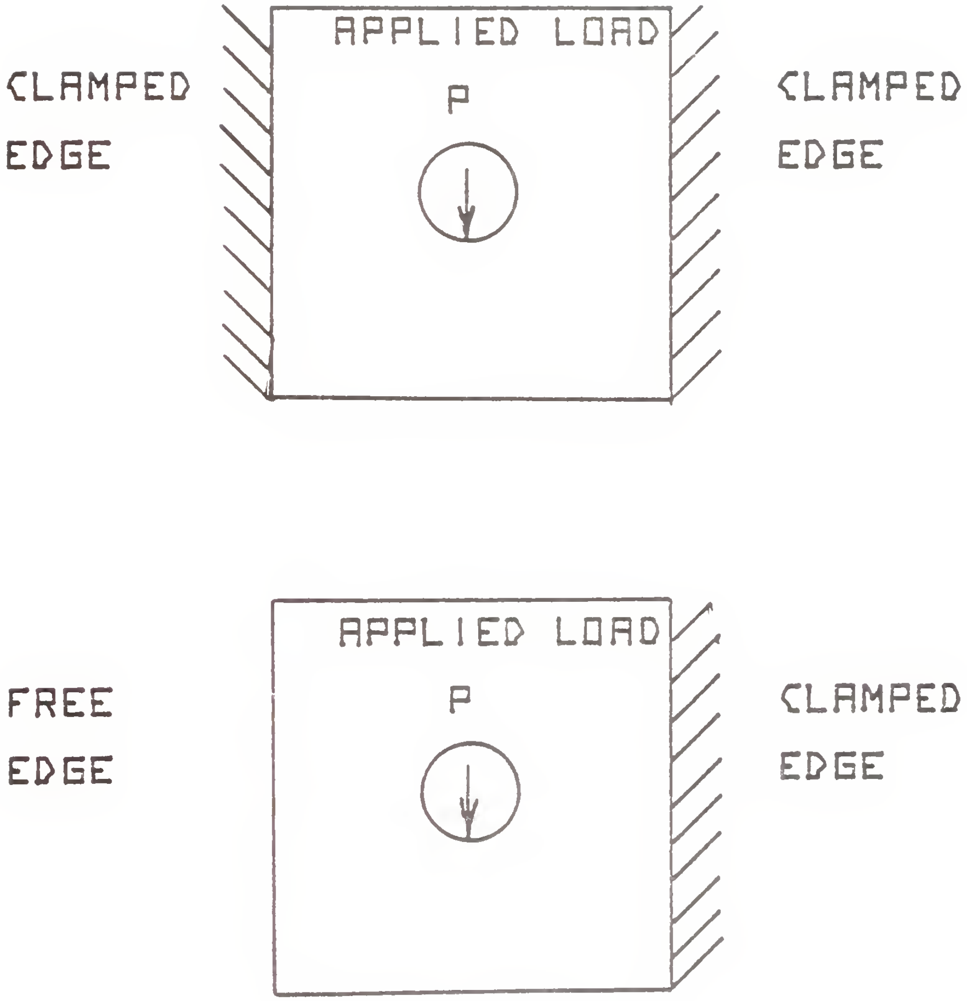
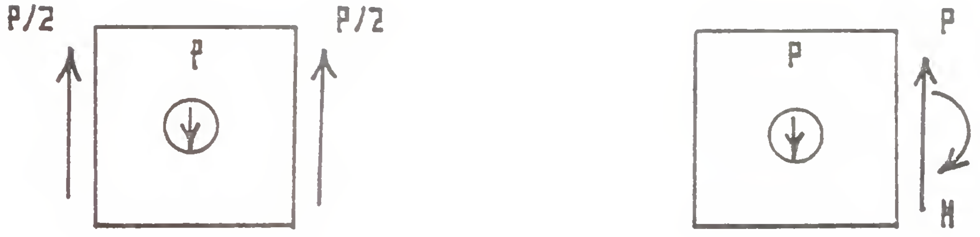


FIGURE 40. SCHEMATIC OF SHEAR LOADING TEST SPECIMENS



TWO SIDES CLAMPED   ONE SIDE CLAMPED

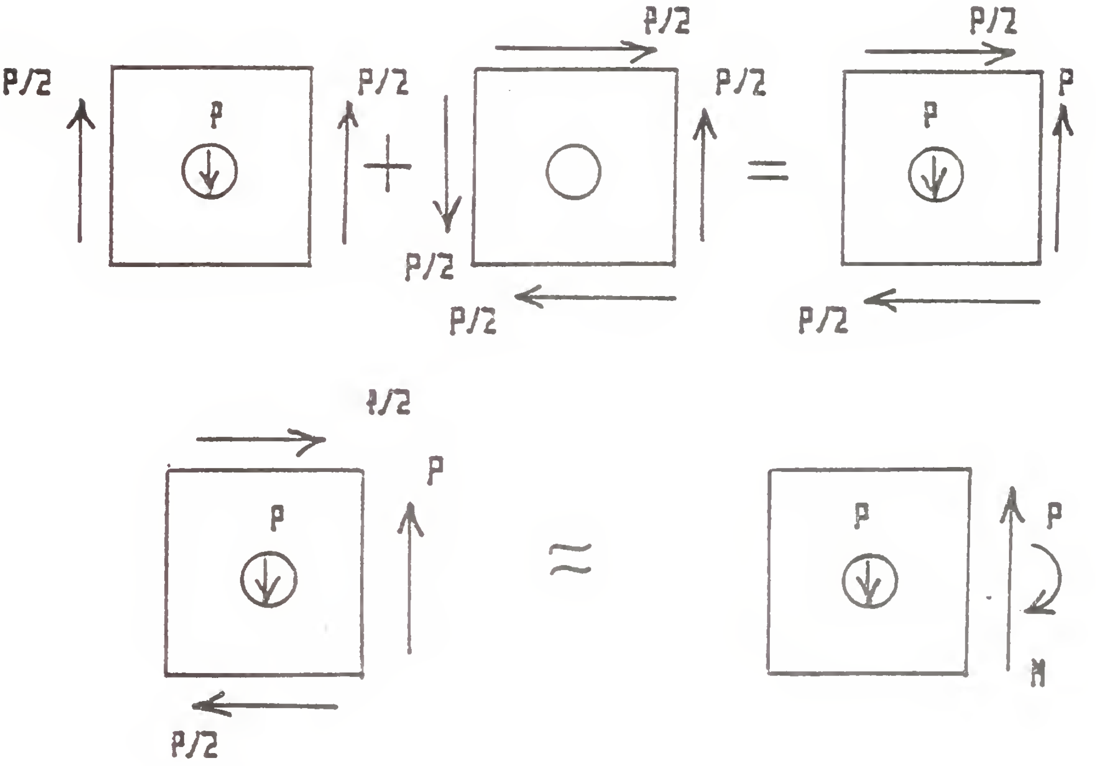


FIGURE 41. SCHEMATIC OF THE SUPERPOSITION USED TO DETERMINE SHEAR EFFECTS ON A BUFFER STRIP WITH A CENTRAL HOLE

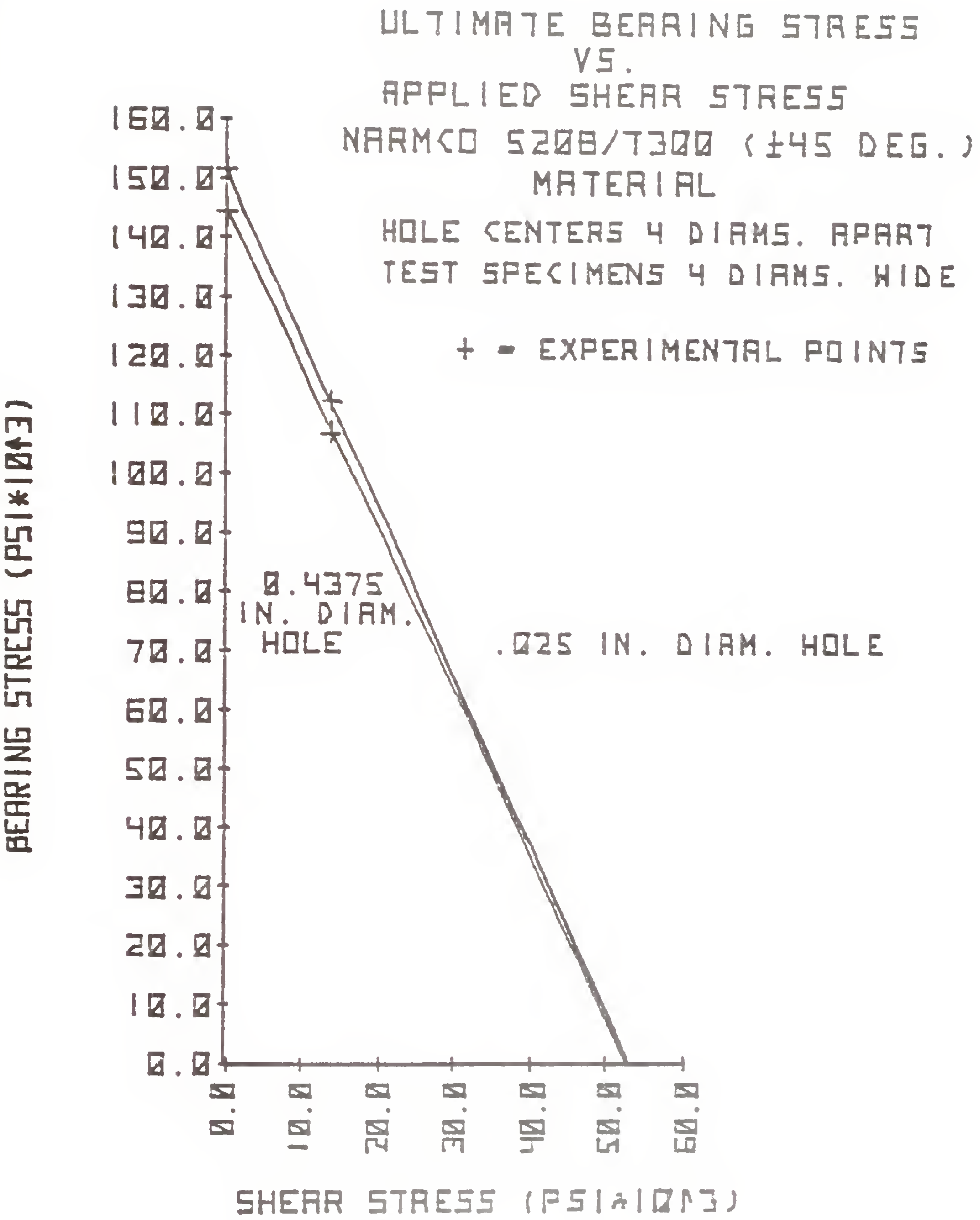


FIGURE 42. ULTIMATE BEARING STRESS-SHEAR STRESS INTERACTION CURVE FOR A FOUR HOLE DIAMETER SQUARE PLATE OF NARMCO 5208/T300 [±45] MATERIAL WITH A CENTRAL HOLE

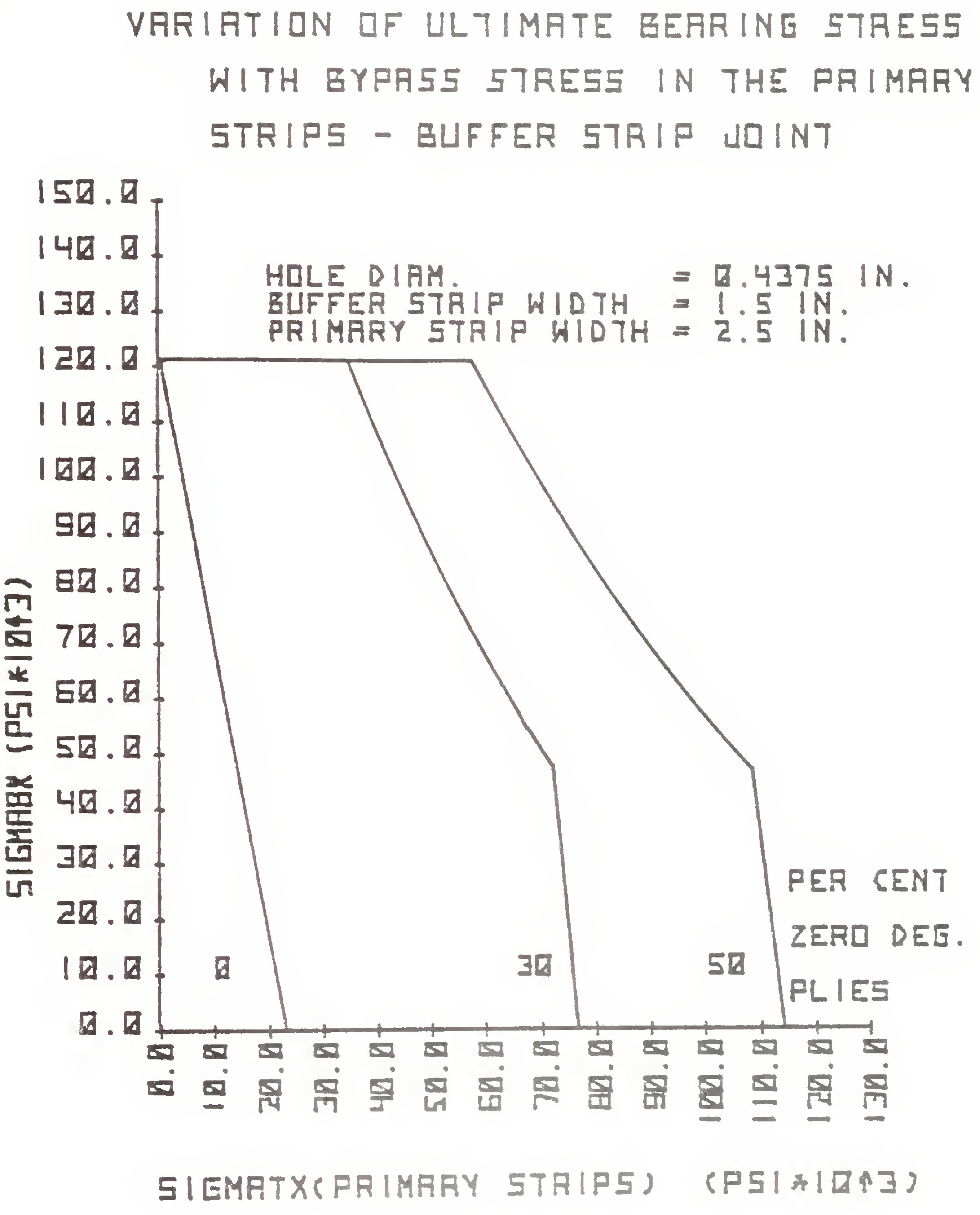


FIGURE 43. ULTIMATE STRESS INTERACTION CURVE FOR A BUFFER STRIP JOINT MADE FROM NARMCO 5208/T300 [0/±45] MATERIAL WITH 2.5 IN. WIDE PRIMARY STRIPS, A 1.5 IN. WIDE BUFFER STRIP, AND A 0.4375 IN. DIAMETER CENTRAL HOLE

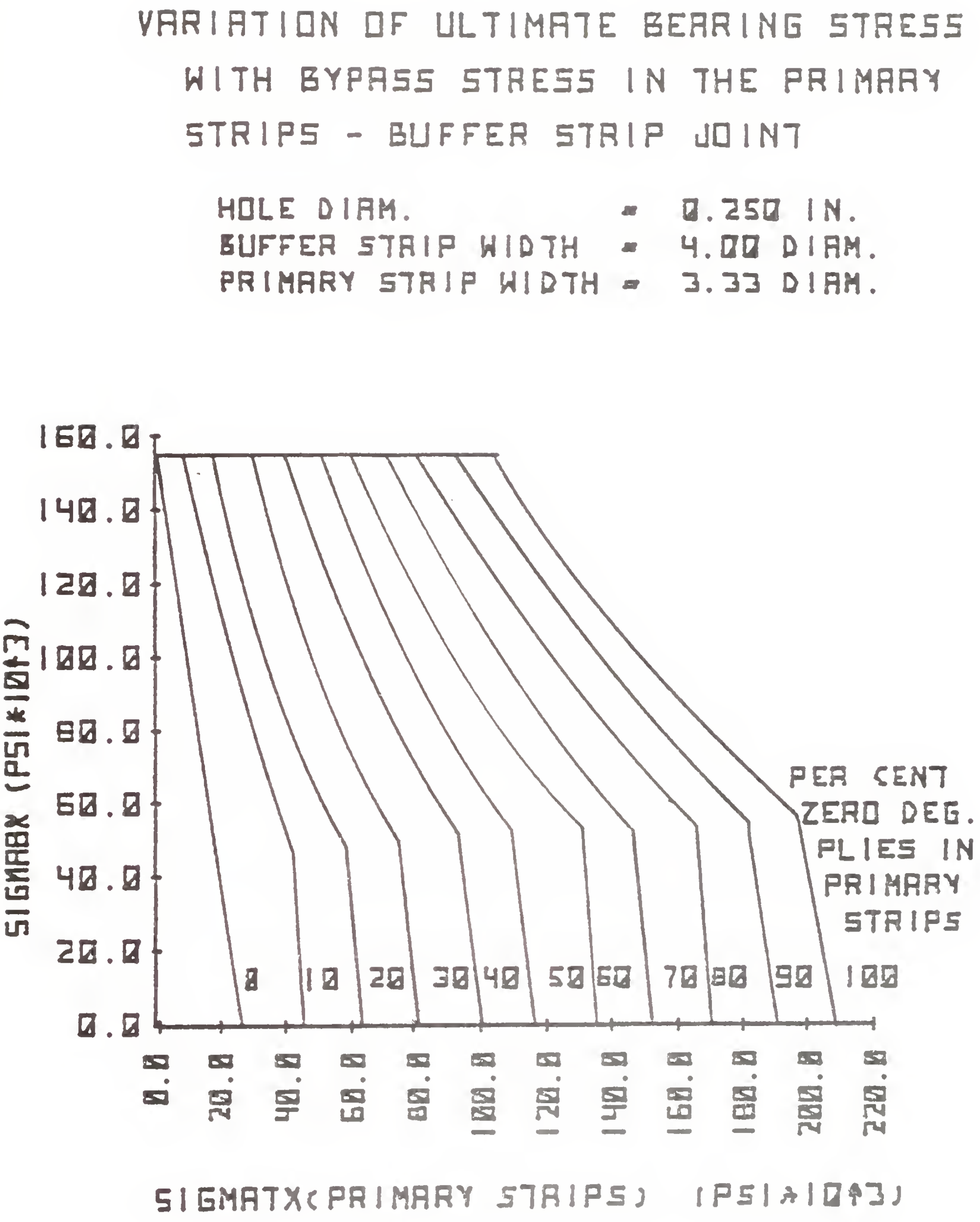


FIGURE 44. ULTIMATE STRESS INTERACTION CURVE FOR A 1.0 IN. LONG BUFFER STRIP PLATE MADE FROM NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH 0.833 IN. WIDE PRIMARY STRIPS, A 1.0 IN. WIDE BUFFER STRIP, AND A 0.25 IN. DIAMETER CENTRAL HOLE

VARIATION OF ULTIMATE BEARING STRESS WITH BYPASS STRESS IN THE PRIMARY STRIPS - BUFFER STRIP JOINT

HOLE DIAM. = 0.4375 IN.
BUFFER STRIP WIDTH = 4.00 DIAM.
PRIMARY STRIP WIDTH = 3.33 DIAM.

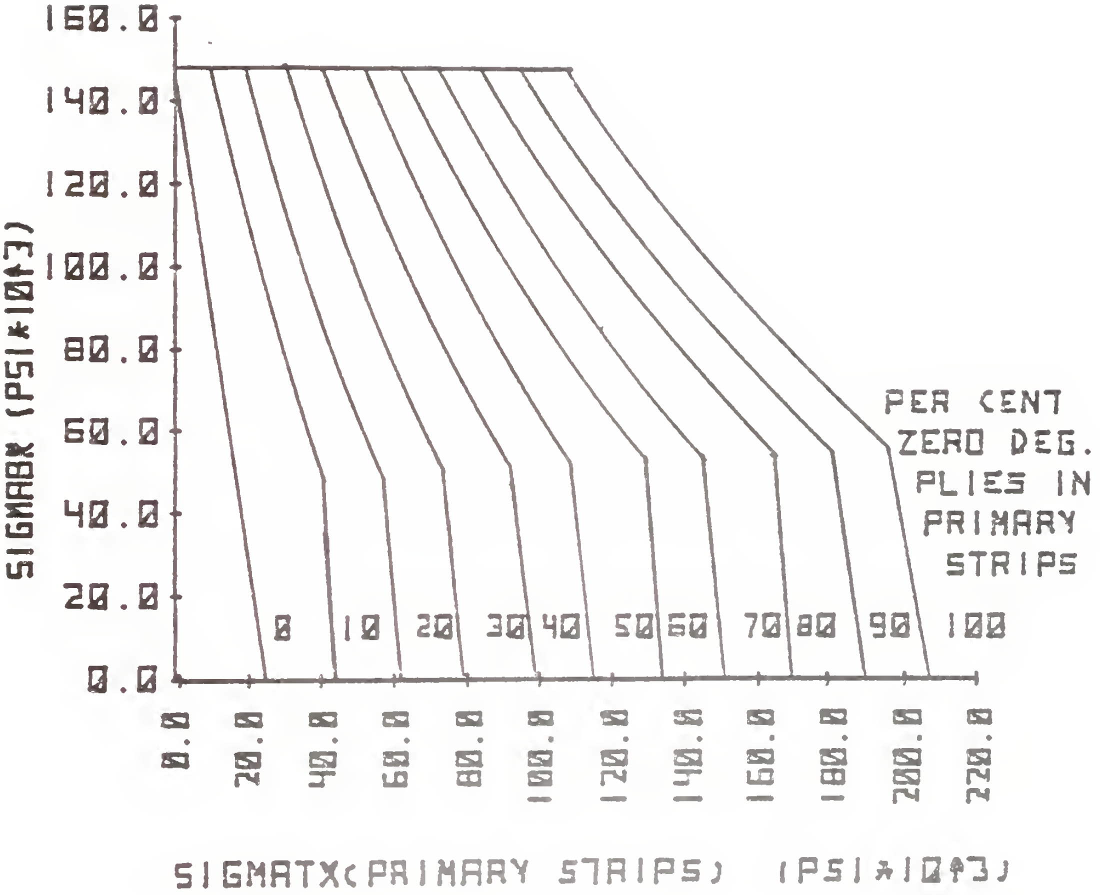


FIGURE 45. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. LONG BUFFER STRIP PLATE MADE FROM NARMCO 5208/T300 [0/±45] MATERIAL WITH 1.46 IN. WIDE PRIMARY STRIPS, A 1.75 IN. WIDE BUFFER STRIP, AND A 0.4375 IN. DIAMETER CENTRAL HOLE

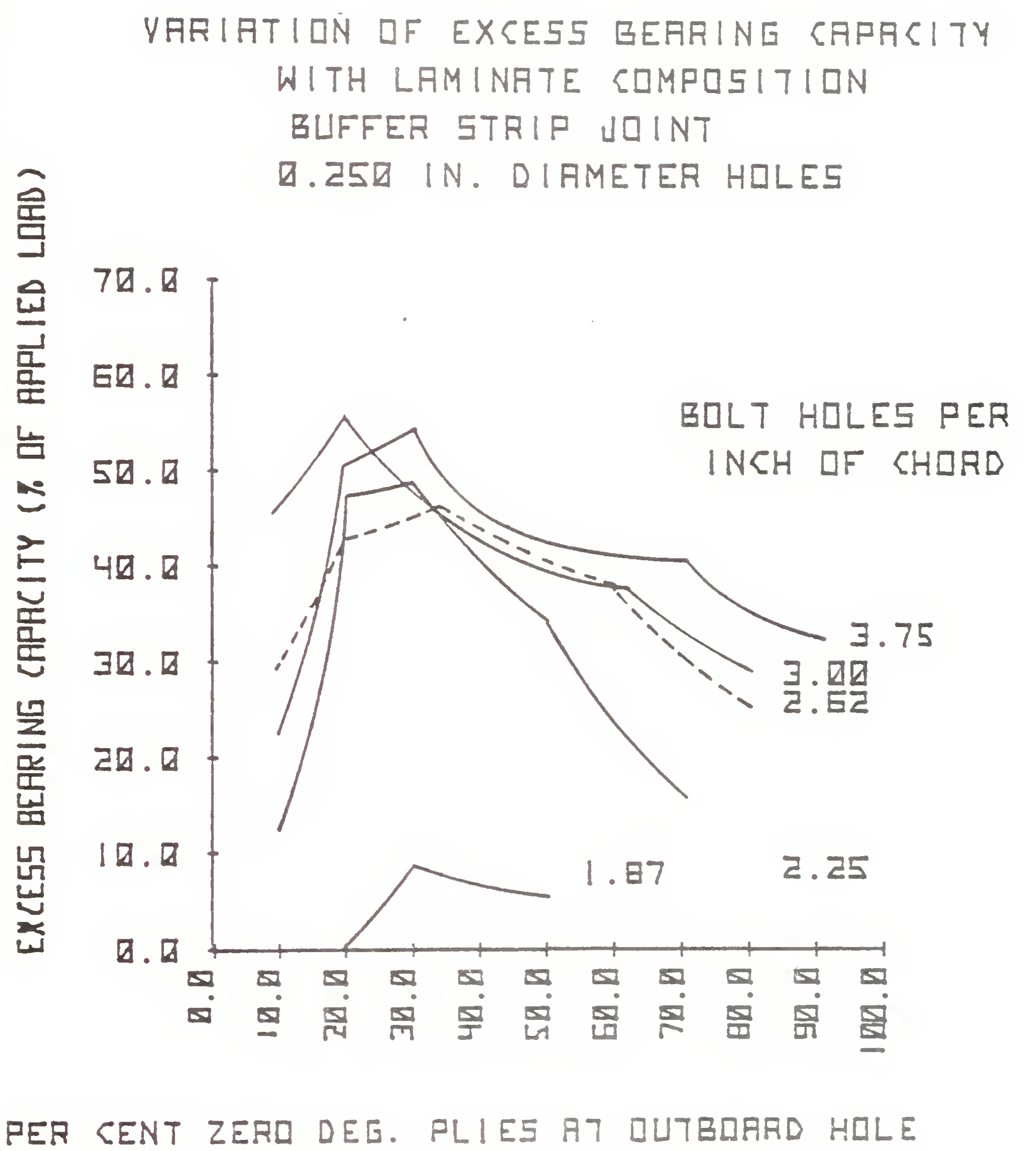


FIGURE 46. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER HOLES

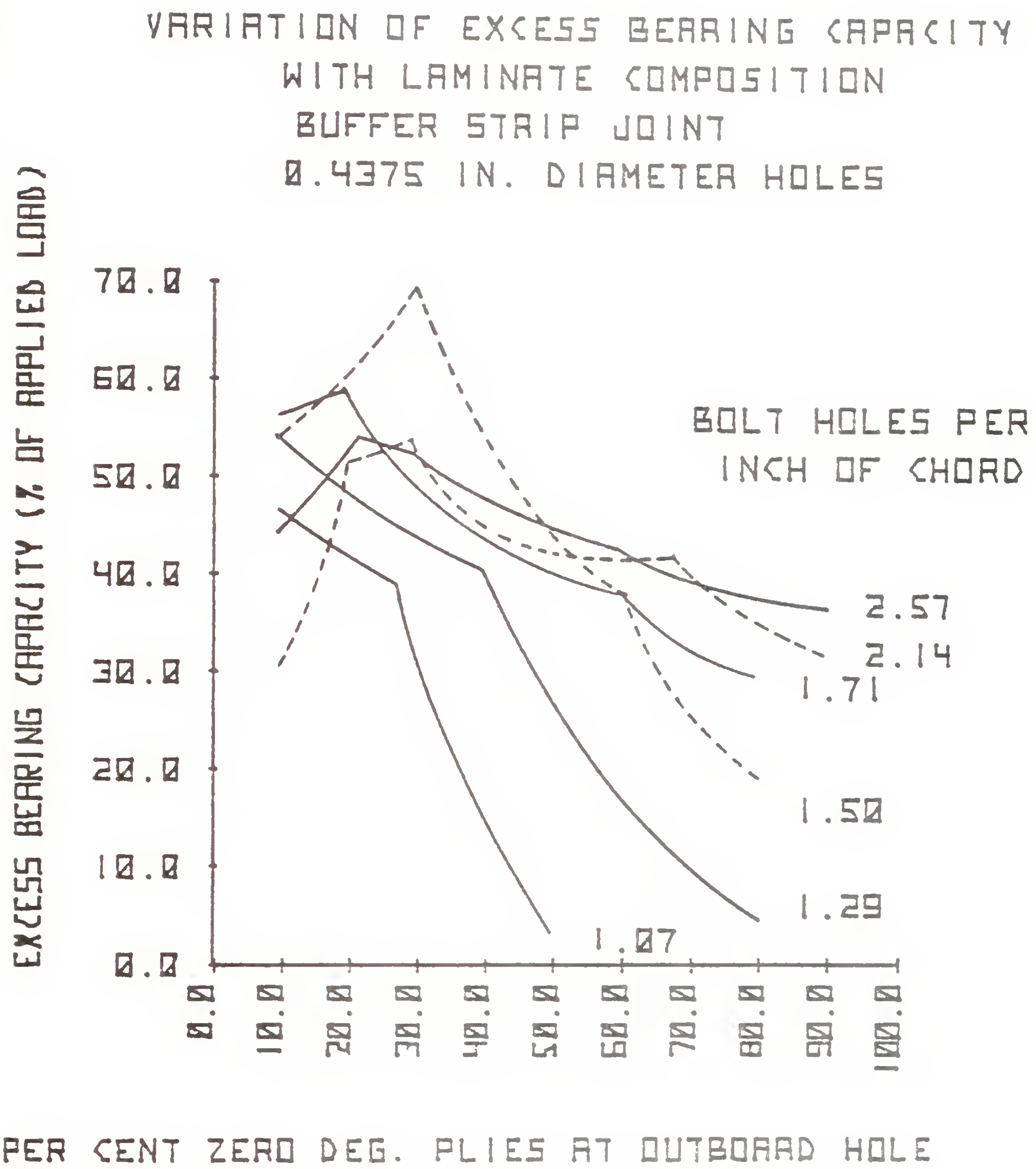


FIGURE 47. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.4375 IN. DIAMETER HOLES

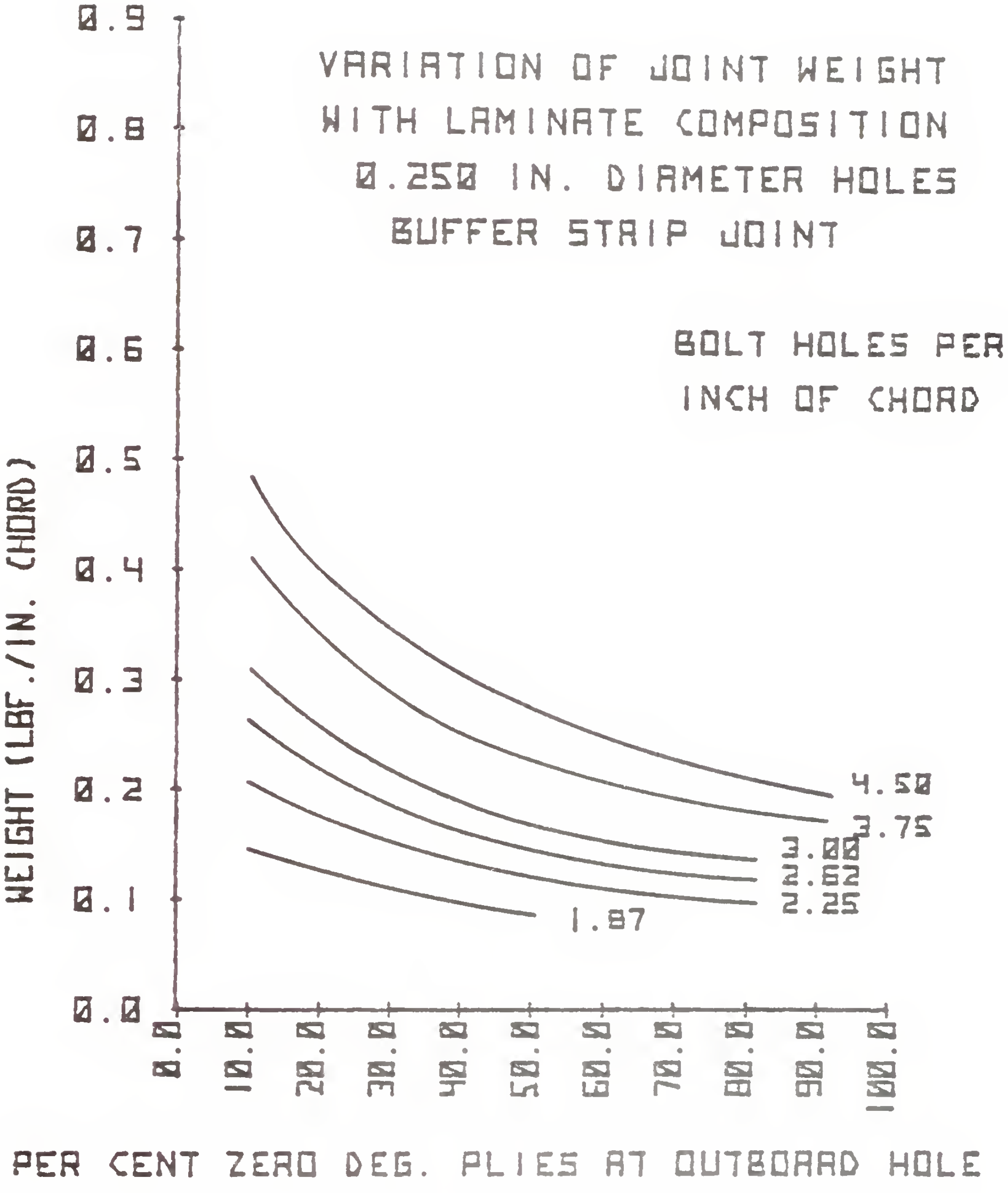


FIGURE 48. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER HOLES

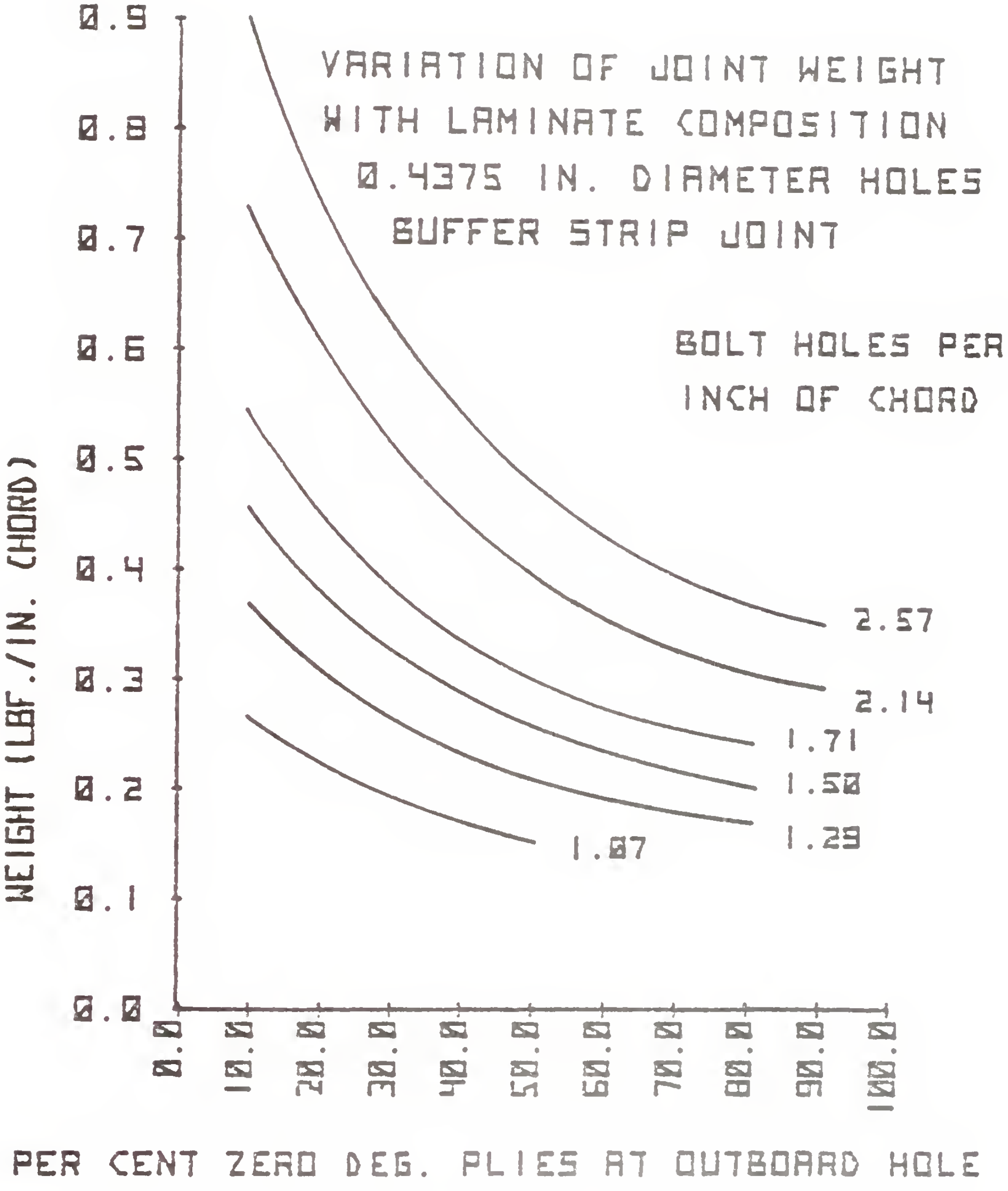


FIGURE 49. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.4375 IN. DIAMETER HOLES

TABLE I

SUMMARY OF DESIGN CONDITIONS AND ASSUMPTIONS

1. The joints are made from NARMCO 5208/T300 $[0/\pm 45]$ graphite-epoxy laminated material.
2. Skin thickness varies linearly within a joint.
3. All bolt holes in a joint are of the same diameter.
4. The interbolt strain level is 3000 micro-in./in.
5. In the theoretical developments for both buffer strip and non-buffer strip joints it was assumed that only tensile and shear loads were to be carried.
6. Each row of bolts reacts an equal portion of the applied tensile load.
7. The applied shear load is reacted by the inboard row of bolts.
8. The minimum number of rows of bolts in any joint is three.
9. Wing taper is disregarded.
10. The inboard row of bolts is in all ± 45 degree laminate.
11. The maximum joint length is ten inches.
12. In the non-buffer strip joints, there is a four-hole-diameter spacing between adjacent rows of bolt hole centers.
13. In the non-buffer strip joint, the laminate between the inboard and next to inboard bolt holes must contain at least five per cent zero degree plies.
14. In the buffer strip joints, the buffer strip width is

four hole diameters. The primary strips are each 3.335 hole diameters wide.

15. Weight and excess bearing capacity calculations were made for assumed load conditions N_x = 20,000 lbf./in. and N_{xy} = 0.

APPENDIX A

```
C PROGRAM ISANIS                                                        MAIN  10
C   ********************************************************************MAIN  20
C    THIS PROGRAM ANALYZES THE STATE OF STRESS AND STRAIN IN A TWO      MAIN  30
C    DIMENSIONAL BODY COMPOSED OF EITHER ISOTROPIC OR                   MAIN  40
C    ORTHOTROPIC MATERIALS.THE ANALYSIS IS PERFORMED BY THE FINITE      MAIN  50
C    ELEMENT METHOD USING ISOPARAMETRIC ELEMENTS.                       MAIN  60
C    ISANIS IS A MODIFICATION OF PLISOP CODED  BY PROFESSOR             MAIN  70
C    GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL IN 1972.            MAIN  80
C    ISANIS EXPANDS PLISOP TO DEAL WITH ORTHOTROPIC ELEMENTS.           MAIN  90
C    CODED BY JAMES M. GILL, APRIL 1975                                 MAIN 100
C   ********************************************************************MAIN 110
C                                                                       MAIN 120
C   INPUT CARDS NEEDED                                                  MAIN 130
C                                                                       MAIN 140
C    TITLE CARD             (20A4)             ONE CARD PER PROBLEM     MAIN 150
C                                                                       MAIN 160
C    PROBLEM PARAMETERS     (10I5,5X,I5)       ONE CARD PER PROBLEM     MAIN 170
C                                                                       MAIN 180
C     COLUMN           VARIABLE                                         MAIN 190
C       1-5            NEL   NUMBER OF ELEMENTS. NEL MAX. IS 182        MAIN 200
C       6-10           NJT   NUMBER OF JOINTS. (MAX. IS 217)            MAIN 210
C      11-15           NMAT  NUMBER OF MATERIALS. (MAX IS 10)           MAIN 220
C      16-20           NCLOAD  NUMBER OF CONCENTRATED LOADS             MAIN 230
C      21-25           NPBC  NUMBER OF JOINTS WITH BOUNDARY CONDITIONS MAIN 240
C      26-30           NSTRES  ISOTROPIC MATERIAL PROBLEM TYPE. NSTRES MAIN 250
C                               = 0 MEANS ISOTROPIC PLANE STRESS.NSTRESMAIN 260
C                              = 1 MEANS ISOTROPIC PLANE STRAIN.        MAIN 270
C                              FOR PLANE STRAIN, THICKNESS              MAIN 280
C                              WILL BE SET EQUAL TO 1.0                 MAIN 290
C      31-35           NGP   NUMBER OF GAUSS POINTS TO BE USED IN THE  MAIN 300
C                            INTEGRATION. (MAX. IS 5 )                  MAIN 310
C      36-40           NTLD  ISOTROPIC THERMAL LOAD INDICATOR. NTLD =1 MAIN 320
C                            MEANS THERE IS AN ISOTROPIC THERMAL LOAD. MAIN 330
C                            NTLD=0 MEANS NO ISOTROPIC THERMAL LOADS.   MAIN 340
C      41-45           NTBY  BODY FORCE LOAD INDICATOR. NTBY = 1 MEANS MAIN 350
C                            A BODY FORCE LOAD IS PRESENT.NTBY= 0       MAIN 360
C                            MEANS NO BODY FORCE LOADS ARE PRESENT.     MAIN 370
C      46-50           NTIN  INITIAL STRETCH INDICATOR. NTIN = 1 MEANS MAIN 380
C                            INITIAL STRETCH IS TO BE CONSIDERED.       MAIN 390
C                            NTIN=0 MEANS NO INITIAL STRETCH.           MAIN 400
C          IF (NTLD+NTBY+NTIN) IS NOT EQUAL TO ZERO, NJT CARDS WILL BEMAIN 410
C          NEEDED TO INPUT THE CONSISTENT LOAD VECTOR RESULTING FROM    MAIN 420
C          THESE CONDITIONS. PROGRAM ISLOAD CAN BE USED TO GENERATE     MAIN 430
C          THIS VECTOR.                                                 MAIN 440
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C      56-60           NLEL  NUMBER OF LAMINATE TYPES WHOSE MATERIAL     MAIN 450
C                      PROPERTIES ARE TO  BE CALCULATED FROM             MAIN 460
C                      INPUT LAMINA PROPERTIES. (MAX. 50)                MAIN 470
C                                                                        MAIN 480
C    JOINT CARDS     (I5,5X,4G15.8,I5)          NJT CARDS PER PROBLEM   MAIN 490
C     COLUMN                VARIABLE                                     MAIN 500
C       1-5                 JOINT NUMBER                                 MAIN 510
C      11-25                X COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 520
C                           RADIUS VECTOR LENGTH (POLAR COORD.)          MAIN 530
C      26-40                Y-COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 540
C                           POLAR ANGLE IN DEGREES (POLAR COORD.)        MAIN 550
C      41-55                X-COMPONENT OF THE LCAD AT THE JOINT (CART)MAIN 560
C      56-70                Y-COMPONENT OF THE LOAD AT THE JOINT (CART)MAIN 570
C      70-75                COORDINATE SYSTEM INDICATOR. IND. = ZERO IFMAIN 580
C                           THE COORDINATE SYSTEM IS CARTESIAN, IF IND MAIN 590
C                           IS NOT ZERO, THE COORDINATE SYSTEM IS POLARMAIN 600
C                                                                        MAIN 610
C     MATERIAL PROPERTIES CARDS(2I5,7F10.2)      NMAT CARDS PER PROBLEM  MAIN 620
C      CCLUMN          VARIABLE                                          MAIN 630
C       1-05           MATERIAL NUMBER                                   MAIN 640
C       6-10           INDK EQ.0 (ISOTROPIC), EQ.1 (ORTHOTROPIC)         MAIN 650
C      11-20           YOUNG'S MODULUS (ISOTROPIC MATERIAL)              MAIN 660
C                      E(L) (ORTHOTROPIC MATERIAL)                       MAIN 670
C      21-30           POISSON'S RATIO (ISOTROPIC MATERIAL)              MAIN 680
C                      E(T) (ORTHOTROPIC MATERIAL)                       MAIN 690
C      31-40           NOT USED (ISOTROPIC MATERIAL)                     MAIN 700
C                      NU(LT) (ORTHOTROPIC MATERIAL)                     MAIN 710
C      41-50           NOT USED (ISOTROPIC MATERIAL)                     MAIN 720
C                      NU(TL) (ORTHOTROPIC MATERIAL)                     MAIN 730
C                 FOR ORTHOTROPIC MATERIALS, ONLY 3 CF THE ABOVE 4 ENGIN.  MAIN 740
C                 CONSTANTS (E(L),E(T),NU(LT),NU(TL)) NEED BE KNOWN. FOR ANMAIN 750
C                 UNKNOWN CONSTANT ENTER -1.0 IN THE APPROPRIATE COLUMN.   MAIN 760
C      51-60           NOT USED (ISOTROPIC MATERIALS)                    MAIN 770
C                      G(LT) (ORTHOTROPIC MATERIALS)                     MAIN 780
C      61-70           ALPHA   COEFF. OF ISOTROPIC THERMAL EXPANSION     MAIN 790
C                      NOT USED FOR THERMALLY ANISOTROPIC MATERIALS      MAIN 800
C      71-80           MATERIAL THICKNESS (USED FOR ELEMENTS             MAIN 810
C                      WHOSE MATERIAL PROPERTIES ARE NOT TO BE           MAIN 820
C                      CALCULATED FROM INPUT LAMINA PROPERTIES.)         MAIN 830
C                                                                        MAIN 840
C                                                                        MAIN 850
C     ELEMENT CARDS          (15I4,F10.5,I5)    NEL CARDS PER PROBLEM    MAIN 860
C       1-4             ELEMENT IDENTIFICATION NUMBER                    MAIN 870
C       5-52            ELEMENT CONNECTIVITY (COUNTER-CLOCKWISE)         MAIN 880
C      53-56           ELEMENT KIND - USE                                MAIN 890
C                       1  FOR A  FOUR  NODAL POINT ELEMENT              MAIN 900
C                       2  FOR AN EIGHT NODAL POINT ELEMENT              MAIN 910
C                       3  FOR A TWELVE NODAL POINT ELEMENT              MAIN 920
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C      57-60           MATERIAL / LAMINATE IDENTIFICATION NUMBER       MAIN 930
C      61-70           THETA (DEGREES) USED FOR ORTHOTROPIC MATERIALS  MAIN 940
C                      THETA IS THE ANGLE FROM THE PROBLEM BASED  X AXISMAIN 950
C                       TO  THE MATERIAL BASED L AXIS                  MAIN 960
C      71-75           ENTER 1 FOR A LAMINATED ELEMENT WHOSE PROPERTIESMAIN 970
C                       ARE TO BE CALCULATED FROM INPUT LAMINA          MAIN 980
C                       PROPERTIES.                                     MAIN 990
C                                                                       MAIN1000
C  BOUNDARY CONDITION CARDS  (2I5)          NPBC  CARDS PER PROBLEM     MAIN1010
C   COL.   VARIABLE                                                     MAIN1020
C    1-5  JOINT NUMBER                                                  MAIN1030
C    6-10 CODE  1 FOR A ROLLER ON AN X AXIS, DISP. IN Y DIRECT. IS ZERO MAIN1040
C               2 FOR A ROLLER ON AN Y AXIS, DISP. IN X DIRECT. IS ZERO MAIN1050
C               3 FOR A FIXED JOINT, BOTH DISP. CCMPONENTS ARE ZERO     MAIN1060
C               4 FOR A ROLLER ON AN X AXIS, THE DISP. IN THE Y DIRECT. MAIN1070
C                 IS SPECIFIED ON THE JCINT CARD IN COL. 41-50          MAIN1080
C               5 FOR A ROLLER ON A Y AXIS, THE DISP. IN THE X DIRECT.  MAIN1090
C                 IS SPECIFIED ON THE JOINT CARD IN COL. 31-40          MAIN1100
C               6 FIXED JOINT WITH DISP. SPECIFIED IN BOTH X AND Y      MAIN1110
C                 DIRECT. ON THE JOINT CARD IN COL.31-40 AND COL.41-50  MAIN1120
C               7 FOR A ROLLER ON AN AXIS INCLINED ALPHA DEGREES        MAIN1130
C                 WITH RESPECT TO THE X AXIS. THE JOINT CARD MAY        MAIN1140
C                 CONTAIN A TANGENTIAL LOAD IN COL. 31-40 AND THE       MAIN1150
C                 ANGLE ALPHA DEGREES IN COL. 41-50                     MAIN1160
C                                                                       MAIN1170
C   LAMINATE CARD DECK       AT LEAST TWO CARDS PER LAMINATE TYPE       MAIN1180
C   CARC ONE          2(I5)                                             MAIN1190
C     COLUMN        VARIABLE                                            MAIN1200
C       1-5         LAMTP        LAMINATE TYPE NUMBER(MAX.50)           MAIN1210
C       6-10        LYRNO        NO. LAMINAE IN THIS LAMTP (MAX.100)    MAIN1220
C   CARD TWO       4(I3,F8.3,F9.4)                                      MAIN1230
C     COLUMN        VARIABLE                                            MAIN1240
C        1-3        MATERIAL NUMBER                                     MAIN1250
C       4-11        LAMINA THICKNESS                                    MAIN1260
C      12-20        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1270
C      21-23        MATERIAL NUMBER                                     MAIN1280
C      24-31        LAMINA THICKNESS                                    MAIN1290
C      32-40        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1300
C      41-43        MATERIAL NUMBER                                     MAIN1310
C      44-51        LAMINA THICKNESS                                    MAIN1320
C      52-60        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1330
C      61-63        MATERIAL NUMBER                                     MAIN1340
C      64-71        LAMINA THICKNESS                                    MAIN1350
C      72-80        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1360
C   AS MANY OF THESE SECOND TYPE CARDS AS NEEDED TO SPECIFY LYRNO LAYERSMAIN1370
C   MAY BE USED                                                         MAIN1380
C   REPEAT THE SEQUENCE WITH NEW CARD TYPE 1 AND 2 AS NEEDED TO         MAIN1390
C   SPECIFY ALL  NLEL LAMINATE TYPES                                    MAIN1400
```

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C                                                                       MAIN1410
C  CONSISTENT LOAD VECTOR (I10,2G25.16)      NJT CARDS PER PROBLEM      MAIN1420
C   NO CARDS NEEDED IF (NTLD+NTBY+NTIN)=0.                              MAIN1430
C   COL.    VARIABLE                                                    MAIN1440
C    1-10 JOINT NUMBER                                                  MAIN1450
C   11-35 X COMPONENT OF LOAD                                           MAIN1460
C   36-70 Y COMPONENT OF LOAD                                           MAIN1470
C                                                                       MAIN1480
C  AS MANY PROBLEMS AS ONE DESIRES MAY BE STACKED, BUT THE LAST CARD OF MAIN1490
C  A RUN MUST CONTAIN THE WORD    STOP    IN  THE FIRST FOUR COLUMNS    MAIN1500
C                                                                       MAIN1510
C        MAIN  PROGRAM                                                  MAIN1520
C                                                                       MAIN1530
      IMPLICIT REAL*8(A-H,O-Z)                                          MAIN1540
      IMPLICIT INTEGER*2(I-N)                                           MAIN1550
      INTEGER *4LMK,LML,NBAND2                                          MAIN1560
      CCMMON /SOL/ BGK(434,86),ALOAD(434),ABGN                          MAIN1570
      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR                           MAIN1580
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCCN(182,15),NBC(217,2),NSTRMAIN1590
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                         MAIN1600
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCCN(10,7),TITLE(10),ANGLMAIN1610
     1(182),THNP(217)                                                   MAIN1620
      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                        MAIN1630
      CCMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                   MAIN1640
      DIMENSION REACT(218,2), NCODE(217)                                MAIN1650
      EQUIVALENCE (BGK(1,1),REACT(1,1))                                 MAIN1660
      MEL = 182                                                         MAIN1670
      MJT = 217                                                         MAIN1680
      MMT = 10                                                          MAIN1690
      MBD = 86                                                          MAIN1700
      MLEL = 50                                                         MAIN1710
      MLYR = 100                                                        MAIN1720
      PI = 3.14159265359D0                                              MAIN1730
      ZRO = 0.0D0                                                       MAIN1740
    1 NBAND = 0                                                         MAIN1750
      NBAND2 = NBAND                                                    MAIN1760
C                                                                       MAIN1770
      DO 2 I=1,MJT                                                      MAIN1780
    2 NCODE(I) = 0                                                      MAIN1790
C                                                                       MAIN1800
      CALL INPUT                                                        MAIN1810
      NEQ = 2*NJT                                                       MAIN1820
      NPEL = NCON(1,13)*4                                               MAIN1830
C                                                                       MAIN1840
      DC 5 I=1,NEL                                                      MAIN1850
C                                                                       MAIN1860
      DO 3 J=1,NPEL                                                     MAIN1870
    3 LM(J) = NCON(I,J)                                                 MAIN1880
```

```
C                                                                       MAIN1890
      NPELM = NPEL-1                                                    MAIN1900
C                                                                       MAIN1910
      DC 4 K=1,NPELM                                                    MAIN1920
      LMK = LM(K)                                                       MAIN1930
      JK = K+1                                                          MAIN1940
C                                                                       MAIN1950
      DC 4 L=JK,NPEL                                                    MAIN1960
      LML = LM(L)                                                       MAIN1970
    4 NBAND2 = MAXO(NBAND2,IABS(LMK-LML))                               MAIN1980
C                                                                       MAIN1990
    5 CCNTINUE                                                          MAIN2000
C                                                                       MAIN2010
      NEAND = NBAND2                                                    MAIN2020
      NBAND = (NBAND+1)*2                                               MAIN2030
      IF (NBAND.GT.MBD) GO TO 16                                        MAIN2040
      WRITE (6,17) NBAND                                                MAIN2050
C                                                                       MAIN2060
      DO 6 I=1,NEQ                                                      MAIN2070
      ALOAD(I) = ZRO                                                    MAIN2080
C                                                                       MAIN2090
      DO 6 J=1,NBAND                                                    MAIN2100
    6 BGK(I,J) = ZRO                                                    MAIN2110
C                                                                       MAIN2120
      CALL MEFGE (NPEL,NEC)                                             MAIN2130
      CALL BCOND (NBAND)                                                MAIN2140
      CALL LDLT (NEQ,NBAND)                                             MAIN2150
C                                                                       MAIN2160
      DO 7 I=1,NPBC                                                     MAIN2170
      IJT = NBC(I,1)                                                    MAIN2180
    7 NCODE(IJT) = NBC(I,2)                                             MAIN2190
C                                                                       MAIN2200
C                                                                       MAIN2210
      DO 10 I=1,NJT                                                     MAIN2220
      II = I*2-1                                                        MAIN2230
      KODE = NCODE(I)                                                   MAIN2240
      IF (KODE.EQ.7) GO TO 9                                            MAIN2250
      IF ((KODE.EQ.5).OR.(KODE.EQ.6)) GC TO 8                           MAIN2260
      ALOAD(II) = CLOAD(I,1)+ALOAD(II)                                  MAIN2270
    8 II = II+1                                                         MAIN2280
      IF ((KODE.EQ.4).OR.(KODE.EQ.6)) GO TO 10                          MAIN2290
      ALOAD(II) = CLOAD(I,2)+ALOAD(II)                                  MAIN2300
      GO TO 10                                                          MAIN2310
    9 ALF = CLOAD(I,2)*PI/180.0D0                                       MAIN2320
      CCSA = DCOS(ALF)                                                  MAIN2330
      SINA = DSIN(ALF)                                                  MAIN2340
      IIP = II+1                                                        MAIN2350
      A1 = ALOAD(II)*COSA+ALOAD(IIP)*SINA+CLOAD(I,1)                    MAIN2360
```

```
      A2 = ALOAD(IIP)*COSA-ALOAD(II)*SINA                               MAIN2370
      ALOAD(II) = A1                                                    MAIN2380
      ALCAD(IIP) = A2                                                   MAIN2390
   10 CONTINUE                                                          MAIN2400
C                                                                       MAIN2410
      SXL = ZRO                                                         MAIN2420
      SYL = ZRO                                                         MAIN2430
C                                                                       MAIN2440
      DO 12 I=1,NJT                                                     MAIN2450
      IX = 2*I-1                                                        MAIN2460
      IY = IX+1                                                         MAIN2470
      IF (NCODE(I).EQ.7) GO TO 11                                       MAIN2480
      SXL = SXL+ALOAD(IX)                                               MAIN2490
      SYL = SYL+ALOAD(IY)                                               MAIN2500
      GO TO 12                                                          MAIN2510
   11 ALF = CLOAD(I,2)*PI/180.0D0                                       MAIN2520
      COSA = DCOS(ALF)                                                  MAIN2530
      SINA = DSIN(ALF)                                                  MAIN2540
      SXL = SXL+ALOAD(IX)*COSA-ALOAD(IY)*SINA                           MAIN2550
      SYL = SYL+ALOAD(IX)*SINA+ALOAD(IY)*COSA                           MAIN2560
   12 CCNTINUE                                                          MAIN2570
C                                                                       MAIN2580
      WRITE (6,24)                                                      MAIN2590
      WRITE (6,22) SXL,SYL                                              MAIN2600
      CALL SOLV (NEQ,NBAND)                                             MAIN2610
C                                                                       MAIN2620
      DO 13 I=1,NPBC                                                    MAIN2630
C                                                                       MAIN2640
      DC 13 J=1,2                                                       MAIN2650
   13 REACT(I,J) = ZRO                                                  MAIN2660
C                                                                       MAIN2670
      CALL DISPL                                                        MAIN2680
      WRITE (6,19)                                                      MAIN2690
C                                                                       MAIN2700
      DC 14 I=1,NJT                                                     MAIN2710
      II = I*2-1                                                        MAIN2720
      III = II+1                                                        MAIN2730
   14 WRITE (6,18) I,ALOAD(II),ALOAD(III)                               MAIN2740
C                                                                       MAIN2750
      WRITE (6,20)                                                      MAIN2760
C                                                                       MAIN2770
      DC 15 I=1,NPBC                                                    MAIN2780
   15 WRITE (6,18) NBC(I,1),REACT(I,1),REACT(I,2)                       MAIN2790
C                                                                       MAIN2800
      WRITE (6,21)                                                      MAIN2810
      MJP = MJT+1                                                       MAIN2820
      WRITE (6,22) REACT(MJP,1),REACT(MJP,2)                            MAIN2830
      CALL STRESS (NPEL)                                                MAIN2840
```

```
      GO TO 1                                                           MAIN2850
  16  WRITE (6,23) MBD,NBAND                                            MAIN2860
      STOP                                                              MAIN2870
C                                                                       MAIN2880
  17  FORMAT (1X,'PROBLEM BANDWIDTH EQUALS ',I5)                        MAIN2890
  18  FORMAT (2X,I3,2G15.4)                                             MAIN2900
  19  FORMAT (//,' DISPLACEMENTS',//,3X,' JOINT',1X,'  U',12X,'  V',/)  MAIN2910
  20  FORMAT (//,'      REACTIONS',//,3X,' JOINT',1X,' RX',12X,' RY',/) MAIN2920
  21  FCRMAT (/,' SUM OF THE REACTIONS AT THE SUPPORTS',/)              MAIN2930
  22  FORMAT (5X,2G15.4)                                                MAIN2940
  23  FORMAT (5X,' MAX. BAND (',I13,') EXCEEDED, NBAND = ',I14)         MAIN2950
  24  FCRMAT (/,'  SUM OF THE EXTERNAL LOADS',/)                        MAIN2960
      END                                                               MAIN2970
      SUBROUTINE INPUT                                                  INPT  10
C  THIS SUBROUTINE READS INPUT DATA FOR THE PROBLEM                     INPT  20
      IMPLICIT REAL*8(A-H,O-Z)                                          INPT  30
      IMPLICIT INTEGER*2(I-N)                                           INPT  40
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRINPT  50
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                         INPT  60
      CCMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR                           INPT  70
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLINPT  80
     1(182),THNP(217)                                                   INPT  90
      CCMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                        INPT 100
      CCMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                   INPT 110
      DATA CHK/'STOP    '/                                              INPT 120
      ZRO = 0.0D0                                                       INPT 130
      PI = 3.14159265359D0                                              INPT 140
      READ (5,30) TITLE                                                 INPT 150
      WRITE (6,31) TITLE                                                INPT 160
      IF (TITLE(1).EQ.CHK) STOP                                         INPT 170
C                                                                       INPT 180
      DO 1 I=1,MJT                                                      INPT 190
      THNP(I) = ZRO                                                     INPT 200
C                                                                       INPT 210
      DO 1 J=1,2                                                        INPT 220
      NBC(I,J) = 0                                                      INPT 230
      COARD(I,J) = ZRO                                                  INPT 240
    1 CLCAD(I,J) = ZRO                                                  INPT 250
C                                                                       INPT 260
C                                                                       INPT 270
      DC 2 I=1,MMT                                                      INPT 280
      INDK(I) = 0                                                       INPT 290
C                                                                       INPT 300
      DC 2 J=1,7                                                        INPT 310
    2 ELCON(I,J) = ZRO                                                  INPT 320
C                                                                       INPT 330
C                                                                       INPT 340
      DO 3 I=1,MEL                                                      INPT 350
```

```
      ANGL(I) = ZRO                                                     INPT 360
C                                                                       INPT 370
      DO 3 J=1,15                                                       INPT 380
    3 NCON(I,J) = 0                                                     INPT 390
C                                                                       INPT 400
C                                                                       INPT 410
      DO 4 I=1,MLEL                                                     INPT 420
      LYRNO(I) = 0                                                      INPT 430
C                                                                       INPT 440
      DO 4 J=1,MLYR                                                     INPT 450
      LAMAT(I,J) = 0                                                    INPT 460
      ORLAM(I,J) = ZRO                                                  INPT 470
    4 TKLAM(I,J) = ZRO                                                  INPT 480
C                                                                       INPT 490
C                                                                       INPT 500
C     READ PROBLEM PARAMETERS                                           INPT 510
C                                                                       INPT 520
      READ (5,32) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN,NLEINPT 530
     1L                                                                 INPT 540
      IF (NEL.GT.MEL) GO TO 23                                          INPT 550
      IF (NJT.GT.MJT) GO TO 24                                          INPT 560
      IF (NMAT.GT.MMT) GO TO 25                                         INPT 570
      IF (NCLOAD.GT.MJT) GO TO 26                                       INPT 580
      IF (NPBC.GT.MJT) GO TO 27                                         INPT 590
      IF (NLEL.GT.MLEL) GO TO 28                                        INPT 600
      WRITE (6,33) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NLEL             INPT 610
C                                                                       INPT 620
C     READ COORDINATES OF JOINTS AND CONCENTRATED LOADS                 INPT 630
C                                                                       INPT 640
      WRITE (6,34)                                                      INPT 650
C                                                                       INPT 660
      DO 5 I=1,NJT                                                      INPT 670
      READ (5,35) IJT,COARD(IJT,1),COARD(IJT,2),CLOAD(IJT,1),CLOAD(IJT,2INPT 680
     1),IND                                                             INPT 690
      IF (IND.EQ.0) GO TO 5                                             INPT 700
      ALF = COARD(IJT,2)*PI/180.0D0                                     INPT 710
      COSA = DCOS(ALF)                                                  INPT 720
      SINA = DSIN(ALF)                                                  INPT 730
      XC = COARD(IJT,1)*COSA                                            INPT 740
      YC = COARD(IJT,1)*SINA                                            INPT 750
      COARD(IJT,1) = XC                                                 INPT 760
      COARD(IJT,2) = YC                                                 INPT 770
    5 CONTINUE                                                          INPT 780
C                                                                       INPT 790
C                                                                       INPT 800
      DO 6 I=1,NJT                                                      INPT 810
    6 WRITE (6,36) I,COARD(I,1),COARD(I,2),CLOAD(I,1),CLOAD(I,2)        INPT 820
C                                                                       INPT 830
```

```
C                                                                       INPT 840
C     READ MATERIAL PROPERTIES CARDS                                    INPT 850
C                                                                       INPT 860
C                                                                       INPT 870
      DO 7 I=1,NMAT                                                     INPT 880
    7 READ (5,37) IMAT,INDK(IMAT),(ELCON(IMAT,J),J=1,7)                 INPT 890
C                                                                       INPT 900
      WRITE (6,38)                                                      INPT 910
C                                                                       INPT 920
      DC 8 I=1,NMAT                                                     INPT 930
      WRITE (6,39) I,(ELCON(I,J),J=1,7),INDK(I)                         INPT 940
    8 CONTINUE                                                          INPT 950
C                                                                       INPT 960
C                                                                       INPT 970
C     READ  THE ELEMENT CARDS                                           INPT 980
C                                                                       INPT 990
      WRITE (6,40)                                                      INPT1000
      WRITE (6,41)                                                      INPT1010
C                                                                       INPT1020
      DO 9 I=1,NEL                                                      INPT1030
    9 READ (5,42) IJT,(NCON(IJT,J),J=1,14),ANGL(IJT),NCON(IJT,15)       INPT1040
C                                                                       INPT1050
C                                                                       INPT1060
      DO 10 I=1,NEL                                                     INPT1070
   10 WRITE (6,43) I,(NCON(I,J),J=1,14),ANGL(I),NCON(I,15)              INPT1080
C                                                                       INPT1090
C                                                                       INPT1100
C     READ BOUNDARY CONDITION CARDS                                     INPT1110
C                                                                       INPT1120
      WRITE (6,44)                                                      INPT1130
C                                                                       INPT1140
      DO 11 I=1,NPBC                                                    INPT1150
      READ (5,45) (NBC(I,J),J=1,2)                                      INPT1160
   11 WRITE (6,46) (NBC(I,J),J=1,2)                                     INPT1170
C                                                                       INPT1180
C                                                                       INPT1190
      DC 12 I=1,NPBC                                                    INPT1200
C                                                                       INPT1210
      DO 12 J=I,NPBC                                                    INPT1220
      IF (NBC(I,1).LE.NBC(J,1)) GO TO 12                                INPT1230
      NT1 = NBC(I,1)                                                    INPT1240
      NT2 = NBC(I,2)                                                    INPT1250
      NBC(I,1) = NBC(J,1)                                               INPT1260
      NBC(I,2) = NBC(J,2)                                               INPT1270
      NBC(J,1) = NT1                                                    INPT1280
      NBC(J,2) = NT2                                                    INPT1290
   12 CONTINUE                                                          INPT1300
C                                                                       INPT1310
```

```
      NSV = 0
C
      DO 13 I=1,NPBC
      IF (NBC(I,2).NE.7) GO TO 13
      NSV = NSV+1
   13 CCNTINUE
C
      NPBCM = NPBC-1
      IF (NSV.EQ.0) GO TO 18
C
      DO 16 I=1,NSV
C
      DC 15 J=1,NPBCM
      IF (NBC(J,2).NE.7) GO TO 15
      NT1 = NBC(J,1)
C
      DO 14 K=J,NPBCM
      KP = K+1
      NBC(K,1) = NBC(KP,1)
   14 NBC(K,2) = NBC(KP,2)
C
      NBC(NPBC,1) = NT1
      NBC(NPBC,2) = 7
      GO TO 16
   15 CCNTINUE
C
   16 CONTINUE
C
      WRITE (6,44)
C
      DC 17 I=1,NPBC
   17 WRITE (6,46) (NBC(I,J),J=1,2)
C
   18 CCNTINUE
      IF (NLEL.EQ.0) GO TO 21
C
C     READ LAMINATE CARD DECK
C
C
      DO 19 I=1,NLEL
      READ (5,47) LAMTP,LYRNO(LAMTP)
      L = LYRNO(LAMTP)
      IF (L.GT.MLYR) GO TO 29
      READ (5,48) ((LAMAT(LAMTP,J),TKLAM(LAMTP,J),ORLAM(LAMTP,J)),J=1,L)
   19 CONTINUE
C
      WRITE (6,49)
C
```

```
      DC 20 I=1,NLEL                                                    INPT1800
      WRITE (6,50) I,LYRNO(I)                                           INPT1810
      L = LYRNO(I)                                                      INPT1820
C                                                                       INPT1830
      DO 20 J=1,L                                                       INPT1840
   20 WRITE (6,51) J,LAMAT(I,J),TKLAM(I,J),ORLAM(I,J)                   INPT1850
C                                                                       INPT1860
C                                                                       INPT1870
C     REAC THE CONSISTENT LOAD VECTOR (IF REQUIRED)                     INPT1880
C                                                                       INPT1890
   21 IF ((NTLD+NTBY+NTIN).EQ.0) RETURN                                 INPT1900
C                                                                       INPT1910
      DO 22 I=1,NJT                                                     INPT1920
      READ (5,52) IJT,CLX,CLY,THNP(I)                                   INPT1930
      CLOAD(IJT,1) = CLOAD(IJT,1)+CLX                                   INPT1940
      CLOAD(IJT,2) = CLOAD(IJT,2)+CLY                                   INPT1950
   22 CCNTINUE                                                          INPT1960
C                                                                       INPT1970
      RETURN                                                            INPT1980
   23 WRITE (6,53) MEL,NEL                                              INPT1990
      STOP                                                              INPT2000
   24 WRITE (6,54) MJT,NJT                                              INPT2010
      STOP                                                              INPT2020
   25 WRITE (6,55) MMT,NMAT                                             INPT2030
      STOP                                                              INPT2040
   26 WRITE (6,56) MJT,NCLOAD                                           INPT2050
      STOP                                                              INPT2060
   27 WRITE (6,57) MJT,NPBC                                             INPT2070
      STOP                                                              INPT2080
   28 WRITE (6,58) MLEL,NLEL                                            INPT2090
      STOP                                                              INPT2100
   29 WRITE (6,59) MLYR,L                                               INPT2110
      STOP                                                              INPT2120
C                                                                       INPT2130
   30 FCRMAT (10A8)                                                     INPT2140
   31 FCRMAT (1X,10A8)                                                  INPT2150
   32 FCRMAT (10I5,5X,I5)                                               INPT2160
   33 FORMAT (//,' NUMBER OF ELEMENTS=',I5,//,' TCTAL NUMBER OF JOINTS='INPT2170
     1,I5,//,' NUMBER OF MATERIALS=',I5,//,' NUMBER OF CONCENTRATED LOADINPT2180
     2S=',I5,//,' NUMBER OF JOINTS WITH BOUNDARY CONDITIONS=',I5,//,' THINPT2190
     3E VALUE OF NSTRES=',I5,//,' NUMBER OF GAUSS PCINTS USED ',I5,//,' INPT2200
     4NUMBER OF LAMINATE TYPES USED',I5)                                INPT2210
   34 FCRMAT (///,' JOINT NUMBER',5X,'X COORDINATE',5X,'Y COORDINATE',7XINPT2220
     1,'X LOAD',9X,' Y LOAD'//)                                         INPT2230
   35 FORMAT (I5,5X,4G15.8,I5)                                          INPT2240
   36 FCRMAT (5X,I3,10X,G14.5,3X,G14.5,5X,G14.5,3X,G14.5)               INPT2250
   37 FCRMAT (2I5,7F10.2)                                               INPT2260
   38 FORMAT (///,' MATERIAL PROPERTIES',//,1X,'MATERIAL    ',2X,'E,E(L)INPT2270
```

```
     1',7X,'NU,E(LT)',5X,'NU(LT)',8X,'NU(TL)',10X,'G(LT)',12X,'ALPHA',9XINPT2280
     2,'THICKNESS',9X,'INDK')                                           INPT2290
   39 FORMAT (' ',1X,I3,5X,7(2X,G13.6),2X,I5)                           INPT2300
   40 FORMAT (///,' ELEMENT CONNECTIVITY AND MATERIAL CARDS')           INPT2310
   41 FORMAT (//,' ELEMENT NUMBER',20X,'JOINT NUMBERS',30X,'KIND',3X,'MAINPT2320
     1TERIAL',3X,'THETA(DEG.)',3X,'LAM. IND.')                          INPT2330
   42 FORMAT (15I4,F10.5,I5)                                            INPT2340
   43 FORMAT (6X,I3,6X,12I5,1X,I5,5X,I5,1X,F10.5,5X,I5)                 INPT2350
   44 FORMAT (///,' BOUNDARY CONDITION CODE',//,5X,'JNT NO',6X,'CODE'//)INPT2360
   45 FORMAT (2I5)                                                      INPT2370
   46 FORMAT (5X,I5,I10)                                                INPT2380
   47 FORMAT (2I5)                                                      INPT2390
   48 FORMAT (4(I3,F8.3,F9.4))                                          INPT2400
   49 FORMAT (///,'LAMINATE PROPERTIES',//,1X,'LAMINATE TYPE',5X,'NO OF INPT2410
     1LAYERS    ',1X,'LAYER NUMBER',5X,'MATERIAL NO.',5X,'THICKNESS',5X,'INPT2420
     2 ORIENTATION'//)                                                  INPT2430
   50 FORMAT (4X,I5,11X,I5)                                             INPT2440
   51 FORMAT (30X,I5,8X,I7,10X,F9.3,5X,F9.3)                            INPT2450
   52 FORMAT (5X,1I5,2G25.16,1G15.8)                                    INPT2460
   53 FORMAT (5X,' MAX. NUMBER (',1I3,') OF ELEMENTS EXCEEDED, NEL=',1I4INPT2470
     1)                                                                 INPT2480
   54 FORMAT (5X,' MAX. NUMB. (',1I3,') OF JOINTS EXCEEDED, NJT =',1I4) INPT2490
   55 FORMAT (5X,' MAX. NUMB. (',1I2,') OF MATERIAL EXCEEDED, NMAT=',1I4INPT2500
     1)                                                                 INPT2510
   56 FORMAT (5X,' MAX. NUMB. (',1I3,') OF C. LOADS EXC., NCLOAD =',1I4)INPT2520
   57 FORMAT (5X,' MAX. N. (',1I3,') OF JOINTS WITH BOUND. COND. EXCDED,INPT2530
     1   NPBC =',1I4)                                                   INPT2540
   58 FORMAT (5X,'MAX. NUMB.(',I5,') OF LAMINATE TYPES EXCEEDED,NLEL =',INPT2550
     1I5)                                                               INPT2560
   59 FORMAT (5X,'MAX. NUMB.(',I5,') OF LAMINATE LAYERS EXCEEDED,LYRNO =INPT2570
     1',I5)                                                             INPT2580
      END                                                               INPT2590
      SUBROUTINE MERGE (NPEL,NEQ)                                       MERG  10
      IMPLICIT REAL*8(A-H,O-Z)                                          MERG  20
      IMPLICIT  INTEGER*2(I-N)                                          MERG  30
      COMMON /SOL/ BGK(434,86),ALOAD(434),ABGN                          MERG  40
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRMERG  50
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                         MERG  60
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLMERG  70
     1(182),THNP(217)                                                   MERG  80
      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                   MERG  90
      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                        MERG 100
      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK              MERG 110
      DIMENSION STK(24,24), AK(24,24), B(3,24)                          MERG 120
      REWIND 10                                                         MERG 130
      ZRO = 0.0D0                                                       MERG 140
C                                                                       MERG 150
      DO 1 I=1,36                                                       MERG 160
```

```
C                                                                       MERG 170
      DO 1 J=1,24                                                       MERG 180
      SS(I,J) = ZRO                                                     MERG 190
    1 SN(I,J) = ZRO                                                     MERG 200
C                                                                       MERG 210
C                                                                       MERG 220
      DO 6 IK=1,NEL                                                     MERG 230
C                                                                       MERG 240
      DO 2 NN=1,NPEL                                                    MERG 250
      LJT(NN) = NCON(IK,NN)                                             MERG 260
    2 LM(NN) = (NCON(IK,NN)-1)*2                                        MERG 270
C                                                                       MERG 280
      NS = 3*NPEL                                                       MERG 290
C                                                                       MERG 300
      DO 3 I1=1,NPEL                                                    MERG 310
      I2 = LJT(I1)                                                      MERG 320
C                                                                       MERG 330
      DO 3 J1=1,2                                                       MERG 340
    3 COORD(I1,J1) = COARD(I2,J1)                                       MERG 350
C                                                                       MERG 360
      INDIC = NCON(IK,15)                                               MERG 370
      IF (INDIC.NE.1) GO TO 4                                           MERG 380
      CALL LAMC (IK)                                                    MERG 390
      GO TO 5                                                           MERG 400
    4 THETA = ANGL(IK)                                                  MERG 410
      MATN = NCON(IK,14)                                                MERG 420
      INDIC = INDK(MATN)                                                MERG 430
      CALL STFMAT (ELAST,MATN,THETA,INDIC,NSTRES)                       MERG 440
      TK = ELCON(MATN,7)                                                MERG 450
    5 N = NCON(IK,13)*8                                                 MERG 460
C    SET THICKNESS =1. FOR PLANE STRAIN                                 MERG 470
      IF (NSTRES.EQ.1) TK=1.0D0                                         MERG 480
      CALL QUAD5 (STK,AK,B,NS,N,NGP)                                    MERG 490
      WRITE (10) SS,SN                                                  MERG 500
      IF (TK.EQ.0.0D0) TK=1.0D0                                         MERG 510
C                                                                       MERG 520
      DO 6 I=1,NPEL                                                     MERG 530
C                                                                       MERG 540
      DO 6 J=1,NPEL                                                     MERG 550
C                                                                       MERG 560
      DO 6 K=1,2                                                        MERG 570
      II = LM(I)+K                                                      MERG 580
      KK = 2*(I-1)+K                                                    MERG 590
C                                                                       MERG 600
      DO 6 L=1,2                                                        MERG 610
      JJ = LM(J)+L-II+1                                                 MERG 620
      IF (JJ.LE.0) GO TO 6                                              MERG 630
      LL = 2*(J-1)+L                                                    MERG 640
```

```
      BGK(II,JJ) = BGK(II,JJ)+STK(KK,LL)*TK                              MERG 650
    6 CONTINUE                                                           MERG 660
C                                                                        MERG 670
      SUM = ZRO                                                          MERG 680
      CON = ZRO                                                          MERG 690
C                                                                        MERG 700
      DO 7 I=1,NEQ                                                       MERG 710
    7 SUM = SUM+BGK(I,1)                                                 MERG 720
C                                                                        MERG 730
      ABGN = SUM*1.0D20                                                  MERG 740
      RETURN                                                             MERG 750
      END                                                                MERG 760
      SUBROUTINE LAMC (IK)                                               LAMC  10
C    THIS SUBROUTINE CALCULATES THE STIFFNESS MATRIX FOR LAMINATED       LAMC  20
C    COMPOSITE MATERIALS. THIS SUBROUTINE ASSUMES THAT THE LAMINATED     LAMC  30
C    COMPOSITE IS OF BALANCED DESIGN SO THAT THE STIFFNESS AND BENDING   LAMC  40
C    PROPERTIES ARE UNCOUPLED.IT CALCULATES AN AVERAGE STIFFNESS MATRIX  LAMC  50
C    FROM THE STIFFNESS PROPERTIES OF EACH LAMINA.                       LAMC  60
C    IK IS THE ELEMENT NUMBER                                            LAMC  70
      IMPLICIT REAL*8(A-H,O-Z)                                           LAMC  80
      IMPLICIT INTEGER*2(I-N)                                            LAMC  90
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRLAMC 100
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                          LAMC 110
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLLAMC 120
     1(182),THNP(217)                                                    LAMC 130
      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                         LAMC 140
      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                    LAMC 150
      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK               LAMC 160
      DIMENSION C(3,3)                                                   LAMC 170
      LAMTP = NCON(IK,14)                                                LAMC 180
      THETA = ANGL(IK)                                                   LAMC 190
      TK = 0.D0                                                          LAMC 200
C                                                                        LAMC 210
      DO 1 I=1,3                                                         LAMC 220
C                                                                        LAMC 230
      DO 1 J=1,3                                                         LAMC 240
    1 C(I,J) = 0.D0                                                      LAMC 250
C                                                                        LAMC 260
      LYRS = LYRNO(LAMTP)                                                LAMC 270
C                                                                        LAMC 280
      DO 3 I=1,LYRS                                                      LAMC 290
      MATN = LAMAT(LAMTP,I)                                              LAMC 300
      PHI = THETA+ORLAM(LAMTP,I)                                         LAMC 310
      INDIC = INDK(MATN)                                                 LAMC 320
      CALL STFMAT (ELAST,MATN,PHI,INDIC,NSTRES)                          LAMC 330
      TK = TK+TKLAM(LAMTP,I)                                             LAMC 340
C                                                                        LAMC 350
      DO 2 J=1,3                                                         LAMC 360
```

```
C                                                                       LAMC 370
      DC 2 K=1,3                                                        LAMC 380
    2 C(J,K) = C(J,K)+ELAST(J,K)*TKLAM(LAMTP,I)                         LAMC 390
C                                                                       LAMC 400
    3 CCNTINUE                                                          LAMC 410
C                                                                       LAMC 420
C                                                                       LAMC 430
      DO 4 J=1,3                                                        LAMC 440
C                                                                       LAMC 450
      DO 4 K=1,3                                                        LAMC 460
    4 ELAST(J,K) = C(J,K)/TK                                            LAMC 470
C                                                                       LAMC 480
      RETURN                                                            LAMC 490
      END                                                               LAMC 500
      SLBROUTINE STFMAT (C,MATN,THETA,INDIC,NSTRES)                     STFM  10
C   THIS SUBROUTINE ASSEMBLES THE ELASTIC STIFFNESS MATRIX FOR A HOOKEAN STFM  20
C   ELEMENT.THE SUBROUTINE CALCULATES AND STORES IN C(3X3)THE VALUES    STFM  30
C   OF THE ELEMENTS OF THIS MATRIX FOR ISOTROPIC MATERIALS UNDER        STFM  40
C   EITHER PLANE STRAIN OR PLANE STRESS OR FOR ANISOTROPIC MATERIALS    STFM  50
C   EXHIBITING TWO DIMENSIONAL ORTHOTROPY.FOR ANISOTROPIC MATERIALS,    STFM  60
C   THE ELEMENTS OF THIS MATRIX ARE FIRST COMPUTED IN THE MATERIAL      STFM  70
C   BASEC L-T COORDINATE SYSTEM AND THEN TRANSFORMED TO THE PROBLEM     STFM  80
C   BASED X-Y COORDINATE SYSTEM.                                        STFM  90
      IMPLICIT REAL*8(A-H,O-Z)                                          STFM 100
      IMPLICIT INTEGER*2(I-N)                                           STFM 110
      CCMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLSTFM 120
     1(182),THNP(217)                                                   STFM 130
      DIMENSION A(3,3), C(3,3)                                          STFM 140
C                                                                       STFM 150
      DC 1 I=1,3                                                        STFM 160
C                                                                       STFM 170
      DO 1 J=1,3                                                        STFM 180
      A(I,J) = 0.0D0                                                    STFM 190
      C(I,J) = 0.0D0                                                    STFM 200
    1 CCNTINUE                                                          STFM 210
C                                                                       STFM 220
      IF (INCIC.EQ.1) GO TO 3                                           STFM 230
      IF (NSTRES.EQ.1) GO TO 2                                          STFM 240
C    FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRESS       STFM 250
C                                                                       STFM 260
      E = ELCCN(MATN,1)                                                 STFM 270
      XNU = ELCON(MATN,2)                                               STFM 280
      EO = E/(1.0D0-XNU*XNU)                                            STFM 290
      C(1,1) = EO                                                       STFM 300
      C(1,2) = EO*XNU                                                   STFM 310
      C(2,1) = C(1,2)                                                   STFM 320
      C(2,2) = C(1,1)                                                   STFM 330
      C(3,3) = EO*(1.0D0-XNU)/2.0D0                                     STFM 340
```

```
      RETURN
    2 CCNTINUE
C
C   FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRAIN
C
      E = ELCCN(MATN,1)
      XNU =  ELCON(MATN,2)
      E1 = E/((1.0D0+XNU)*(1.0D0-2.0D0*XNU))
      C(1,1) = E1*(1.0D0-XNU)
      C(1,2) = E1*XNU
      C(2,1) = C(1,2)
      C(2,2) = C(1,1)
      C(3,3) = E1*(0.5D0-XNU)
      RETURN
    3 CCNTINUE
C
C   FORM THE ANISOTROPIC ORTHOTROPIC ELASTIC MATRIX IN THE MATERIAL BASED
C   L-T CCORDINATE SYSTEM
C
      EL = ELCON(MATN,1)
      ET = ELCON(MATN,2)
      XNULT = ELCON(MATN,3)
      XNUTL = ELCON(MATN,4)
      GLT = ELCON(MATN,5)
      IF (EL.LT.-0.0D0) EL=XNULT*ET/XNUTL
      IF (ET.LT.-0.0D0) ET=EL*XNUTL/XNULT
      IF (XNULT.LT.-0.0D0) XNULT=EL*XNUTL/ET
      IF (XNUTL.LT.-0.0D0) XNUTL=XNULT*ET/EL
      Z = 1.0D0-XNULT*XNUTL
      A(1,1) = EL/Z
      A(1,2) = ET*XNULT/Z
      A(2,1) = A(1,2)
      A(2,2) = ET/Z
      A(3,3) = GLT
C
C    THIS ARRAY IS NOW TRANSFORMED FROM THE MATERIAL L-T COORDINATE
C    SYSTEM TO THE PROBLEM X-Y COORDINATE SYSTEM.THE TRANSFORMATION IS
C    ACCCMPLISHED BY ROTATION ABOUT THE Z AXIS THRCUGH THETA DEGREES.
C
      PI = 3.14159265359D0
      THRAD = THETA*PI/180.0D0
      CCS1 = DCOS(THRAD)
      CCS2 = DCOS(THRAD)**2
      CCS3 = DCOS(THRAD)**3
      COS4 = DCOS(THRAD)**4
      SIN1 = DSIN(THRAD)
      SIN2 = DSIN(THRAD)**2
      SIN3 = CSIN(THRAD)**3
```

```
      SIN4 = DSIN(THRAD)**4
      TWO = 2.0D0
      FOUR = 4.0D0
      C(1,1) = A(1,1)*COS4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*SIN4
      C(1,2) = (A(1,1)+A(2,2)-FOUR*A(3,3))*SIN2*CCS2+A(1,2)*(SIN4+COS4)
      C(1,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN1*COS3+(A(1,2)-A(2,2)+TWO*(
     1A(3,3)))*SIN3*COS1
      C(2,2) = A(1,1)*SIN4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*COS4
      C(2,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN3*COS1+(A(1,2)-A(2,2)+TWO*A
     1(3,3))*SIN1*COS3
      C(3,3) = (A(1,1)+A(2,2)-TWO*(A(1,2)+A(3,3)))*SIN2*COS2+A(3,3)*(SIN
     14+COS4)
      C(2,1) = C(1,2)
      C(3,1) = C(1,3)
      C(3,2) = C(2,3)
      RETURN
      END
      SUBROUTINE QUAD5 (STK,AK,B,NS,N,NGP)
C
C    QUAD5 IS A NUMERICAL INTEGRATION SUBROUTINE THAT FORMS THE STIFFNESS
C    MATRIX FOR ANY OF THE THREE QUADRILATERAL ISOPARAMETRIC
C    ELEMENTS USED IN THIS PROGRAM.
C    GAUSSIAN QUADRATURE WITH A MAXIMUM OF FIVE ORDINATES IS USED.
C
C        STK  IS AN ARRAY DIMENSIONED NXN THAT WILL CONTAIN THE STIFFNES
C             MATRIX UPON RETURN TO THE CALLING PROGRAM
C        SS AND SN ARE THE STRESS AND STRAIN VERSUS NODAL POINT
C             DISPLACEMENT TRANSFORMATION MATRICES, BOTH ARE DIMENSIONED
C             NSXN WHERE NS= 3*NPT AND N= 2*NPT
C        AK   IS AN ARRAY DIMENSIONED NXN REQUIRED IN FORMK
C        B    IS AN ARRAY DIMENSIONED 2XN REQUIRED IN FORMK
C
C        NGP IS THE NUMBER OF GAUSS POINTS USED IN THE INTEGRATION
C
C        COORD(12,2) IS AN ARRAY CONTAINING THE COORDINATES OF ALL
C                    THE NODAL POINTS OF THE ELEMENT TO BE CONSTRUCTED
C        ELAST(3,3)  IS AN ARRAY CONTAINING ALL THE ELASTIC CONSTANTS
C                    OF THE ELEMENT, THE ORDER OF THE STRESS COMPONENTS
C                    IS: SIGMAX,SIGMAY,TAUXY
C
C
C             ETA                  ETA                  ETA
C              +                    +                    +
C         U2----+-----U1       U3----U2----U1       U4--U3+-U2--U1
C         |     +      |       |      +      |      |      +     |
C         |     +      |       |      +      |      |      +     |
C         |     +      |       |      +      |      |      +     |
C         |     +      |       |      +      |      U5     +    U12
```

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C         |     +     |       |     +     |       |     +     |         QUAD 320
C         |     ++++++++XI    U4    ++++++U8++XI  |     ++++++++XI      QUAD 330
C         |           |       |           |       |           |         QUAD 340
C         |           |       |           |       U6          U11       QUAD 350
C         |           |       |           |       |           |         QUAD 360
C         |           |       |           |       |           |         QUAD 370
C         |           |       |           |       |           |         QUAD 380
C         U3----------U4      U5----U6----U7      U7--U8--U9--U10       QUAD 390
C                                                                       QUAD 400
C   CODED BY PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL  QUAD 410
C                                                                       QUAD 420
      IMPLICIT REAL*8(A-H,O-Z)                                          QUAD 430
      IMPLICIT INTEGER*2(I-N)                                           QUAD 440
      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK              QUAD 450
      DIMENSION AK(24,24), STK(24,24), B(3,24)                          QUAD 460
      DIMENSION XI(5), AI(5), AIA(5,5)                                  QUAD 470
      DIMENSION X2(2), X3(3), X4(4), X5(5), A2(2), A3(3), A4(4), A5(5)  QUAD 480
      DIMENSION XYL(4,2), XYQ(8,2), XYC(12,2)                           QUAD 490
      DATA X2/.577350269189626D0,-.577350269189626D0/                   QUAD 500
      DATA A2/1.0D0,1.0D0/                                              QUAD 510
      DATA X3/.774596669241483D0,0.0D0,-.774596669241483D0/             QUAD 520
      DATA A3/.555555555555556D0,.888888888888889D0,.555555555555556D0/ QUAD 530
      DATA X4/.8611363115940D0,.3399810435848D0,-.3399810435848D0,-.8611QUAD 540
     1363115940D0/                                                      QUAD 550
      DATA A4/.3478548451374D0,.6521451548625D0,.6521451548625D0,.347854QUAD 560
     18451374D0/                                                        QUAD 570
      DATA X5/.9061798459386D0,.5384693101056D0,0.0D0,-.5384693101056D0,QUAD 580
     1-.9061798459386D0/                                                QUAD 590
      DATA A5/.2369268850561D0,.4786286704993D0,.5688888888888D0,.478628QUAD 600
     16704993D0,.2369268850561D0/                                       QUAD 610
      DATA XYL/1.0D0,-1.0D0,-1.0D0,1.0D0,1.0D0,1.0D0,-1.0D0,-1.0D0/     QUAD 620
      DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,QUAD 630
     11.0D0,1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0/                     QUAD 640
      DATA XYC/1.0D0,.3333333333333D0,-.3333333333333D0,-1.0D0,-1.0D0,-1QUAD 650
     1.0D0,-1.0D0,-.3333333333333D0,.3333333333333D0,1.0D0,1.0D0,1.0D0,1QUAD 660
     2.0D0,1.0D0,1.0D0,1.0D0,.3333333333333D0,-.3333333333333D0,-1.0D0,-QUAD 670
     31.0D0,-1.0D0,-1.0D0,-.3333333333333D0,.3333333333333D0/           QUAD 680
      IF (NGP.EQ.0) NGP = 3                                             QUAD 690
C                                                                       QUAD 700
      DO 1 I=1,N                                                        QUAD 710
C                                                                       QUAD 720
      DO 1 J=1,N                                                        QUAD 730
    1 STK(I,J) = 0.0D0                                                  QUAD 740
C                                                                       QUAD 750
      JUMP = NGP-1                                                      QUAD 760
C                                                                       QUAD 770
      DO 6 I=1,NGP                                                      QUAD 780
      GO TO (2,3,4,5), JUMP                                             QUAD 790
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```
    2 XI(I) = X2(I)                               QUAD 800
      AI(I) = A2(I)                               QUAD 810
      GO TO 6                                     QUAD 820
    3 XI(I) = X3(I)                               QUAD 830
      AI(I) = A3(I)                               QUAD 840
      GC TO 6                                     QUAD 850
    4 XI(I) = X4(I)                               QUAD 860
      AI(I) = A4(I)                               QUAD 870
      GO TO 6                                     QUAD 880
    5 XI(I) = X5(I)                               QUAD 890
      AI(I) = A5(I)                               QUAD 900
    6 CCNTINUE                                    QUAD 910
C                                                 QUAD 920
C                                                 QUAD 930
      DO 7 I=1,NGP                                QUAD 940
C                                                 QUAD 950
      DC 7 J=1,NGP                                QUAD 960
    7 AIA(I,J) = AI(I)*AI(J)                      QUAD 970
C                                                 QUAD 980
C                                                 QUAD 990
      DO 8 I=1,NGP                                QUAD1000
      X = XI(I)                                   QUAD1010
C                                                 QUAD1020
      DC 8 J=1,NGP                                QUAD1030
      Y = XI(J)                                   QUAD1040
      CALL FORMK (AK,X,Y,B,N)                     QUAD1050
      AIAIJ = AIA(I,J)                            QUAD1060
C                                                 QUAD1070
      DC 8 K=1,N                                  QUAD1080
C                                                 QUAD1090
      DO 8 L=1,N                                  QUAD1100
    8 STK(K,L) = STK(K,L)+AIAIJ*AK(K,L)           QUAD1110
C                                                 QUAD1120
C                                                 QUAD1130
      DC 9 I=1,N                                  QUAD1140
C                                                 QUAD1150
      DO 9 J=I,N                                  QUAD1160
      STK(I,J) = (STK(I,J)+STK(J,I))*0.5D0        QUAD1170
    9 STK(J,I) = STK(I,J)                         QUAD1180
C                                                 QUAD1190
      NPT = N/2                                   QUAD1200
      IGO = NPT/4                                 QUAD1210
C                                                 QUAD1220
      DG 16 I=1,NPT                               QUAD1230
      GO TO (10,11,12), IGO                       QUAD1240
   10 X = XYL(I,1)                                QUAD1250
      Y = XYL(I,2)                                QUAD1260
      GC TO 13                                    QUAD1270
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   11 X = XYQ(I,1)                                                        QUAD1280
      Y = XYQ(I,2)                                                        QUAD1290
      GC TO 13                                                            QUAD1300
   12 X = XYC(I,1)                                                        QUAD1310
      Y = XYC(I,2)                                                        QUAD1320
   13 CALL FORMK (AK,X,Y,B,N)                                             QUAD1330
C                                                                         QUAD1340
      CC 14 L=1,3                                                         QUAD1350
C                                                                         QUAD1360
      DO 14 M=1,N                                                         QUAD1370
      AK(L,M) = 0.0D0                                                     QUAD1380
C                                                                         QUAD1390
      DO 14 NN=1,3                                                        QUAD1400
   14 AK(L,M) = AK(L,M)+ELAST(L,NN)*B(NN,M)                               QUAD1410
C                                                                         QUAD1420
      II = 3*(I-1)                                                        QUAD1430
C                                                                         QUAD1440
      DO 15 J=1,3                                                         QUAD1450
      IJ = II+J                                                           QUAD1460
C                                                                         QUAD1470
      DO 15 K=1,N                                                         QUAD1480
      SS(IJ,K) = AK(J,K)                                                  QUAD1490
   15 SN(IJ,K) = B(J,K)                                                   QUAD1500
C                                                                         QUAD1510
   16 CCNTINUE                                                            QUAD1520
C                                                                         QUAD1530
      RETURN                                                              QUAD1540
      END                                                                 QUAD1550
      SLBROUTINE FORMK (AK,X,Y,B,N)                                       FRMK  10
C  FCRMK FORMS THE STIFFNESS MATRIX AND THE  B MATRIX AS FUNCTIONS        FRMK  20
C  OF XI AND ETA FOR THE ISOPARAMETRIC ELEMENTS USED IN THIS PROGRAM      FRMK  30
C                                                                         FRMK  40
      IMPLICIT REAL*8(A-H,O-Z)                                            FRMK  50
      IMFLICIT INTEGER*2(I-N)                                             FRMK  60
      CIMENSION AJ(2,2), AJIN(2,2), DNX(2,12), W1(2,12), B(3,24), B1(24,FRMK  70
     13), AK(24,24)                                                       FRMK  80
      CCMMCN COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK                FRMK  90
      ZERO = 0.0D0                                                        FRMK 100
C                                                                         FRMK 110
      DO 1 I=1,3                                                          FRMK 120
C                                                                         FRMK 130
      DO 1 J=1,N                                                          FRMK 140
    1 B(I,J) = ZERO                                                       FRMK 150
C                                                                         FRMK 160
      CNE = 1.0D0                                                         FRMK 170
      TWO = 2.0D0                                                         FRMK 180
      FOUR = 4.0D0                                                        FRMK 190
      NFT = N/2                                                           FRMK 200
```

```
C                                                                       FRMK 210
C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA     FRMK 220
C                                                                       FRMK 230
      IGC = N/8                                                         FRMK 240
      GO TO (2,3,4), IGO                                                FRMK 250
C                                                                       FRMK 260
C    LINEAR FUNCTIONS                                                   FRMK 270
C                                                                       FRMK 280
    2 W1(1,3) = -(ONE-Y)/FOUR                                           FRMK 290
      W1(1,4) = -W1(1,3)                                                FRMK 300
      W1(1,1) = (ONE+Y)/FOUR                                            FRMK 310
      W1(1,2) = -W1(1,1)                                                FRMK 320
      W1(2,3) = -(ONE-X)/FOUR                                           FRMK 330
      W1(2,4) = -(CNE+X)/FOUR                                           FRMK 340
      W1(2,1) = -W1(2,4)                                                FRMK 350
      W1(2,2) = -W1(2,3)                                                FRMK 360
      GC TO 5                                                           FRMK 370
    3 CONTINUE                                                          FRMK 380
C                                                                       FRMK 390
C   CUADRATIC FUNCTIONS                                                 FRMK 400
C                                                                       FRMK 410
      TXPY = TWO*X+Y                                                    FRMK 420
      TXMY = TWO*X-Y                                                    FRMK 430
      TYPX = TWO*Y+X                                                    FRMK 440
      TYMX = TWO*Y-X                                                    FRMK 450
      CMY = ONE-Y                                                       FRMK 460
      OPY = ONE+Y                                                       FRMK 470
      OPX = ONE+X                                                       FRMK 480
      OMX = CNE-X                                                       FRMK 490
      W1(1,5) = OMY*TXPY/FOUR                                           FRMK 500
      W1(1,6) = -OMY*X                                                  FRMK 510
      W1(1,7) = OMY*TXMY/FOUR                                           FRMK 520
      W1(1,8) = OPY*OMY/TWO                                             FRMK 530
      W1(1,1) = OPY*TXPY/FOUR                                           FRMK 540
      W1(1,2) = -OPY*X                                                  FRMK 550
      W1(1,3) = OPY*TXMY/FOUR                                           FRMK 560
      W1(1,4) = -OPY*OMY/TWO                                            FRMK 570
      W1(2,5) = OMX*TYPX/FOUR                                           FRMK 580
      W1(2,6) = -OPX*OMX/TWO                                            FRMK 590
      W1(2,7) = OPX*TYMX/FOUR                                           FRMK 600
      W1(2,8) = -Y*OPX                                                  FRMK 610
      W1(2,1) = OPX*TYPX/FOUR                                           FRMK 620
      W1(2,2) = OPX*OMX/TWO                                             FRMK 630
      W1(2,3) = OMX*TYMX/FOUR                                           FRMK 640
      W1(2,4) = -Y*OMX                                                  FRMK 650
      GO TO 5                                                           FRMK 660
    4 CCNTINUE                                                          FRMK 670
C                                                                       FRMK 680
```

```
C     CUBIC FUNCTIONS
C
       CPX = ONE+X
       OPY = ONE+Y
       CMX = ONE-X
       OMY = ONE-Y
       TPTMNX = 3.0D0+2.0D0*X-9.0D0*X*X
       TMTMNX = 3.0D0-2.0D0*X-9.0D0*X*X
       TPTMNY = 3.0D0+2.0D0*Y-9.0D0*Y*Y
       TMTMNY = 3.0D0-2.0D0*Y-9.0D0*Y*Y
       TYPO = 3.0D0*Y+ONE
       TYMO = 3.0D0*Y-ONE
       TXPO = 3.0D0*X+ONE
       TXMO = 3.0D0*X-ONE
       TMTPEX = 10.0D0-27.0D0*X*X+18.0D0*X-9.0D0*Y*Y
       TMTMEX = 10.0D0-27.0D0*X*X-18.0D0*X-9.0D0*Y*Y
       TMNPEY = 10.0D0-9.0D0*X*X+18.0D0*Y-27.0D0*Y*Y
       TMNMEY = 10.0D0-9.0D0*X*X-18.0D0*Y-27.0D0*Y*Y
       TTT = 32.0D0
       TTN = 32.0D0/9.0D0
       W1(1,7) = OMY*TMTPEX/TTT
       W1(1,8) = -TPTMNX*OMY/TTN
       W1(1,9) = TMTMNX*OMY/TTN
       W1(1,10) = -OMY*TMTMEX/TTT
       W1(1,11) = -OPY*OMY*TYMO/TTN
       W1(1,12) = OPY*OMY*TYPO/TTN
       W1(1,1) = -OPY*TMTMEX/TTT
       W1(1,2) = TMTMNX*OPY/TTN
       W1(1,3) = -TPTMNX*OPY/TTN
       W1(1,4) = OPY*TMTPEX/TTT
       W1(1,5) = -OPY*OMY*TYPO/TTN
       W1(1,6) = OPY*OMY*TYMO/TTN
       W1(2,7) = OMX*TMNPEY/TTT
       W1(2,8) = OPX*OMX*TXMO/TTN
       W1(2,9) = -OPX*OMX*TXPO/TTN
       W1(2,10) = OPX*TMNPEY/TTT
       W1(2,11) = -TPTMNY*OPX/TTN
       W1(2,12) = TMTMNY*OPX/TTN
       W1(2,1) = -OPX*TMNMEY/TTT
       W1(2,2) = OPX*OMX*TXPO/TTN
       W1(2,3) = -OPX*OMX*TXMO/TTN
       W1(2,4) = -OMX*TMNMEY/TTT
       W1(2,5) = OMX*TMTMNY/TTN
       W1(2,6) = -OMX*TPTMNY/TTN
     5 CCNTINUE
C
C FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)
C
```

```
C                                                                       FRMK1170
      DC 6 I=1,2                                                        FRMK1180
C                                                                       FRMK1190
      DO 6 J=1,2                                                        FRMK1200
      AJ(I,J) = ZERO                                                    FRMK1210
C                                                                       FRMK1220
      DO 6 K=1,NPT                                                      FRMK1230
    6 AJ(I,J) = AJ(I,J)+W1(I,K)*COORD(K,J)                              FRMK1240
C                                                                       FRMK1250
C                                                                       FRMK1260
C CALCULATE DETERMINANT OF THE JACOBIAN                                 FRMK1270
C                                                                       FRMK1280
      DTJ = AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)                             FRMK1290
C                                                                       FRMK1300
C INVERT JACOBIAN IN AJIN                                               FRMK1310
C                                                                       FRMK1320
      AJIN(1,1) = AJ(2,2)/DTJ                                           FRMK1330
      AJIN(1,2) = -AJ(1,2)/DTJ                                          FRMK1340
      AJIN(2,1) = -AJ(2,1)/DTJ                                          FRMK1350
      AJIN(2,2) = AJ(1,1)/DTJ                                           FRMK1360
C                                                                       FRMK1370
C FCRM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y       FRMK1380
C                                                                       FRMK1390
C                                                                       FRMK1400
      DC 7 I=1,2                                                        FRMK1410
C                                                                       FRMK1420
      DO 7 J=1,NPT                                                      FRMK1430
      DNX(I,J) = ZERO                                                   FRMK1440
C                                                                       FRMK1450
      DC 7 K=1,2                                                        FRMK1460
    7 DNX(I,J) = DNX(I,J)+AJIN(I,K)*W1(K,J)                             FRMK1470
C                                                                       FRMK1480
C                                                                       FRMK1490
C FORM B(I,J) MATRIX                                                    FRMK1500
C                                                                       FRMK1510
C                                                                       FRMK1520
      DC 8 I=1,NPT                                                      FRMK1530
      IOD = 2*I-1                                                       FRMK1540
      IEV = 2*I                                                         FRMK1550
      B(1,IOD) = DNX(1,I)                                               FRMK1560
      B(2,IEV) = DNX(2,I)                                               FRMK1570
      B(3,IEV) = DNX(1,I)                                               FRMK1580
    8 B(3,IOD) = DNX(2,I)                                               FRMK1590
C                                                                       FRMK1600
C                                                                       FRMK1610
C FORM STIFFNESS BY THE CONGRUENT TRANSFORMATION  BT*D*B                FRMK1620
C                                                                       FRMK1630
C                                                                       FRMK1640
```

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      DC 9 I=1,N                                                        FRMK1650
C                                                                       FRMK1660
      DO 9 J=1,3                                                        FRMK1670
      B1(I,J) = ZERO                                                    FRMK1680
C                                                                       FRMK1690
      DO 9 K=1,3                                                        FRMK1700
    9 B1(I,J) = B1(I,J)+B(K,I)*ELAST(K,J)                               FRMK1710
C                                                                       FRMK1720
C                                                                       FRMK1730
      DC 10 I=1,N                                                       FRMK1740
C                                                                       FRMK1750
      DO 10 J=1,N                                                       FRMK1760
      AK(I,J) = ZERO                                                    FRMK1770
C                                                                       FRMK1780
      DC 10 K=1,3                                                       FRMK1790
   10 AK(I,J) = AK(I,J)+B1(I,K)*B(K,J)                                  FRMK1800
C                                                                       FRMK1810
C                                                                       FRMK1820
      DC 11 I=1,N                                                       FRMK1830
C                                                                       FRMK1840
      DC 11 J=I,N                                                       FRMK1850
      AK(I,J) = ((AK(I,J)+AK(J,I))/2.0D0)*DTJ                           FRMK1860
   11 AK(J,I) = AK(I,J)                                                 FRMK1870
C                                                                       FRMK1880
      RETURN                                                            FRMK1890
      END                                                               FRMK1900
      SUBROUTINE LDLT (N,M)                                             LDLT  10
C ***********************************************************************LDLT  20
C   THIS SUBROUTINE DECOMPOSES THE COEFFICIENT MATRIX A* OF THE LINEAR  LDLT  30
C   BANDED SYMMETRICAL SYSTEM (A*)(X) = (B), INTO THE PRODUCT (L)(D)(LT)LDLT  40
C   THE UPPER BAND ONLY IS USED, AND IT MUST BE STORED IN A RECTANGULAR LDLT  50
C   ARRAY (A) WITH ALL ITS DIAGONAL ELEMENTS IN THE FIRST COLUMN        LDLT  60
C   MATRIX (A) IS DESTROYED IN THE PROCESS AND THE RESULTS RETURNED IN  LDLT  70
C   THE SAME ARRAY. MATRIX (D) IS PUT IN THE FIRST COLUMN AND THE REST  LDLT  80
C   OF THE MATRIX CONTAINS THE ELEMENTS OF THE UNIT TRIANGULAR MATRIX   LDLT  90
C   IN BANDED FORM WITHOUT ITS IMPLIED UNIT DIAGONAL ELEMENTS.          LDLT 100
C                                                                       LDLT 110
C        A  IS THE NAME OF THE COEFFICIENT MATRIX                       LDLT 120
C        N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM                    LDLT 130
C        M  IS THE HALF BAND WIDTH OF THE SYSTEM                        LDLT 140
C                                                                       LDLT 150
C   CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972         LDLT 160
C ***********************************************************************LDLT 170
      IMPLICIT REAL*8(A-H,O-Z)                                          LDLT 180
      IMPLICIT INTEGER*2(I-N)                                           LDLT 190
      COMMON /SOL/ A(434,86),B(434),ABGN                                LDLT 200
      AN = N                                                            LDLT 210
      APZRO = ABGN*1.0D-30/AN                                           LDLT 220
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      AVSN = APZRO/ABGN                                                 LDLT 230
      ZRC = 0.0D0                                                       LDLT 240
      NM = N-1                                                          LDLT 250
C                                                                       LDLT 260
      DC 3 I=1,NM                                                       LDLT 270
      DIAG = A(I,1)                                                     LDLT 280
      IF (DIAG.EQ.ZRO) GO TO 4                                          LDLT 290
      IF (DIAG.LE.APZRO) WRITE (6,6) I                                  LDLT 300
C                                                                       LDLT 310
      DO 1 J=2,M                                                        LDLT 320
      IF (DABS(A(I,J)).LE.AVSN) A(I,J)=0.0D0                            LDLT 330
    1 A(I,J) = A(I,J)/DIAG                                              LDLT 340
C                                                                       LDLT 350
C                                                                       LDLT 360
      DC 3 J=2,M                                                        LDLT 370
      L = I+J-1                                                         LDLT 380
      IF (L.GT.N) GO TO 3                                               LDLT 390
      AA = A(I,J)*DIAG                                                  LDLT 400
      IF (AA.EQ.ZRO) GO TO 3                                            LDLT 410
C                                                                       LDLT 420
      DO 2 K=J,M                                                        LDLT 430
      ML = 1+K-J                                                        LDLT 440
    2 A(L,ML) = A(L,ML)-AA*A(I,K)                                       LDLT 450
C                                                                       LDLT 460
    3 CCNTINUE                                                          LDLT 470
C                                                                       LDLT 480
      RETURN                                                            LDLT 490
    4 WRITE (6,5) I                                                     LDLT 500
      STOP                                                              LDLT 510
C                                                                       LDLT 520
    5 FORMAT (//,5X,' MATRIX IS SINGULAR ',1I5)                         LDLT 530
    6 FCRMAT (//,5X,' WARNING  MATRIX IS NEARLY SINGULAR  EXECUTION CONTLDLT 540
     1INUES ', 1I5,/,5X,' RESIDUAL ITERATION MAY YIELD ERRONEOUS RESULTSLDLT 550
     2    '//)                                                          LDLT 560
      END                                                               LDLT 570
      SUBROUTINE SOLV (N,M)                                             SOLV  10
C ***********************************************************************SOLV  20
C    THIS SUBROUTINE PERFORMS A FORWARD SUBSTITUTION, FOLLOWED BY A     SOLV  30
C    BACKWARD SUBSTITUTION FOR THE SYSTEM (A*) X= (B), AFTER THE        SOLV  40
C    MATRIX (A) HAS BEEN DECOMPOSED BY THE SUBROUTINE LDLT              SOLV  50
C                                                                       SOLV  60
C         A  IS THE NAME OF THE DECOMPOSED MATRIX CF COEFFICIENT        SOLV  70
C         B  IS THE NAME OF THE COLUMN LOAD VECTOR OR RIGHT HAND MEMBER SOLV  80
C            THE ANSWERS ARE RETURNED IN THIS VECTCR UPON EXIT          SOLV  90
C         N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM                   SOLV 100
C         M  IS THE HALF BAND WIDTH OF THE SYTEM                        SOLV 110
C                                                                       SOLV 120
C    AFTER ONE CALL TO LDLT,ONE CALL OF SOLV PER LOAD VECTOR IS REQ'D.  SOLV 130
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C     CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972       SOLV 140
C ********************************************************************SOLV 150
      IMPLICIT REAL*8(A-H,O-Z)                                          SOLV 160
      IMPLICIT INTEGER*2(I-N)                                           SOLV 170
      COMMON /SOL/ A(434,86),B(434),ABGN                                SOLV 180
      NM = N-1                                                          SOLV 190
C                                                                       SOLV 200
      DO 1 I=1,NM                                                       SOLV 210
      BI = B(I)                                                         SOLV 220
C                                                                       SOLV 230
      DO 1 J=2,M                                                        SOLV 240
      L = I+J-1                                                         SOLV 250
      IF (L.GT.N) GO TO 1                                               SOLV 260
      B(L) = B(L)-A(I,J)*BI                                             SOLV 270
    1 CONTINUE                                                          SOLV 280
C                                                                       SOLV 290
C                                                                       SOLV 300
      DO 2 I=1,N                                                        SOLV 310
    2 B(I) = B(I)/A(I,1)                                                SOLV 320
C                                                                       SOLV 330
C                                                                       SOLV 340
      DO 3 L=2,N                                                        SOLV 350
      IR = N-L+1                                                        SOLV 360
      BIRP = B(IR+1)                                                    SOLV 370
C                                                                       SOLV 380
      DO 3 J=2,M                                                        SOLV 390
      IRO = IR-J+2                                                      SOLV 400
      IF (IRO.LE.0) GO TO 3                                             SOLV 410
      B(IRO) = B(IRO)-A(IRO,J)*BIRP                                     SOLV 420
    3 CONTINUE                                                          SOLV 430
C                                                                       SOLV 440
      RETURN                                                            SOLV 450
      END                                                               SOLV 460
      SUBROUTINE STRESS (NPEL)                                          STRS  10
      IMPLICIT REAL*8(A-H,O-Z)                                          STRS  20
      IMPLICIT INTEGER*2(I-N)                                           STRS  30
      COMMON /SOL/ BGK(434,86),DISP(434),ABGN                           STRS  40
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRSTRS  50
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                         STRS  60
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLSTRS  70
     1(182),THNP(217)                                                   STRS  80
      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK              STRS  90
      DIMENSION SSJNT(217,7), SNJNT(217,7), SSEL(36), SNEL(36), DSPEL(36STRS 100
     1)                                                                 STRS 110
      DIMENSION TEEL(12)                                                STRS 120
      EQUIVALENCE (BGK(1,1),SSJNT(1,1)), (BGK(1,5),SNJNT(1,1)), (BGK(1,9STRS 130
     1),SSEL(1)), (BGK(1,10),SNEL(1)), (BGK(1,11),DSPEL(1))             STRS 140
      PI = 3.14159265359D0                                              STRS 150
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```
      ZRC = 0.0D0                                 STRS 160
      REWIND 10                                   STRS 170
C                                                 STRS 180
      DC 1 I=1,NJT                                STRS 190
C                                                 STRS 200
      DO 1 J=1,7                                  STRS 210
      SNJNT(I,J) = ZRO                            STRS 220
    1 SSJNT(I,J) = ZRO                            STRS 230
C                                                 STRS 240
C                                                 STRS 250
      DO 11 I=1,NEL                               STRS 260
      READ (10) SS,SN                             STRS 270
      IF (NTLD.EC.0) GO TO 3                      STRS 280
      MATN = NCON(I,14)                           STRS 290
      YMOD = ELCON(MATN,1)                        STRS 300
      PRAT = ELCON(MATN,2)                        STRS 310
      ALPH = ELCON(MATN,3)                        STRS 320
      IF (NSTRES.EQ.0) GO TO 2                    STRS 330
      CCNST = ALPH*YMOD/(1.0D0-2.0D0*PRAT)        STRS 340
      GO TO 3                                     STRS 350
    2 CONST = ALPH*YMOD/(1.0D0-PRAT)              STRS 360
    3 CONTINUE                                    STRS 370
C                                                 STRS 380
      DO 4 J=1,NPEL                               STRS 390
      NCIJ = NCON(I,J)                            STRS 400
      JA = NCIJ*2-1                               STRS 410
      JJ = J*2-1                                  STRS 420
      IF (NTLD.NE.0) TEEL(J)=THNP(NCIJ)           STRS 430
      DSPEL(JJ) = DISP(JA)                        STRS 440
      JJ = JJ+1                                   STRS 450
      JA = JA+1                                   STRS 460
    4 DSPEL(JJ) = DISP(JA)                        STRS 470
C                                                 STRS 480
      NJLM = NPEL*2                               STRS 490
      NPELT = NPEL*3                              STRS 500
C                                                 STRS 510
      DC 5 I1=1,NPELT                             STRS 520
      SSEL(I1) = ZRO                              STRS 530
      SNEL(I1) = ZRO                              STRS 540
C                                                 STRS 550
      DO 5 J1=1,NJLM                              STRS 560
      SSEL(I1) = SSEL(I1)+SS(I1,J1)*DSPEL(J1)     STRS 570
    5 SNEL(I1) = SNEL(I1)+SN(I1,J1)*DSPEL(J1)     STRS 580
C                                                 STRS 590
      IF (NTLD.EQ.0) GO TO 7                      STRS 600
C                                                 STRS 610
      DO 6 J=1,NPEL                               STRS 620
      JJO = J*3-2                                 STRS 630
```

```
      JJD = JJO+1                                                       STRS 640
      CCRT = CONST*TEEL(J)                                              STRS 650
      SSEL(JJO) = SSEL(JJO)-CORT                                        STRS 660
      SSEL(JJD) = SSEL(JJD)-CORT                                        STRS 670
    6 CCNTINUE                                                          STRS 680
C                                                                       STRS 690
    7 CONTINUE                                                          STRS 700
      IF (NMAT.EQ.1) GO TO 9                                            STRS 710
      WRITE (6,19) I                                                    STRS 720
      ILO = 0                                                           STRS 730
      IUP = 0                                                           STRS 740
C                                                                       STRS 750
      DC 8 J=1,NPEL                                                     STRS 760
      IJT = NCON(I,J)                                                   STRS 770
      ILC = IUP+1                                                       STRS 780
      IUP = ILO+2                                                       STRS 790
    8 WRITE (6,18) IJT,(SSEL(K),K=ILO,IUP),(SNEL(L),L=ILO,IUP)          STRS 800
C                                                                       STRS 810
    9 CCNTINUE                                                          STRS 820
      ICT = 0                                                           STRS 830
C                                                                       STRS 840
      DC 11 I2=1,NPEL                                                   STRS 850
      I2A = NCON(I,I2)                                                  STRS 860
C                                                                       STRS 870
      DO 10 J2=1,3                                                      STRS 880
      ICT = ICT+1                                                       STRS 890
      SSJNT(I2A,J2) = SSEL(ICT)+SSJNT(I2A,J2)                           STRS 900
   10 SNJNT(I2A,J2) = SNEL(ICT)+SNJNT(I2A,J2)                           STRS 910
C                                                                       STRS 920
   11 SNJNT(I2A,7) = SNJNT(I2A,7)+1.                                    STRS 930
C                                                                       STRS 940
C                                                                       STRS 950
      DC 15 I3=1,NJT                                                    STRS 960
      SCNT = SNJNT(I3,7)                                                STRS 970
C                                                                       STRS 980
      DC 12 J3=1,3                                                      STRS 990
      SSJNT(I3,J3) = SSJNT(I3,J3)/SCNT                                  STRS1000
   12 SNJNT(I3,J3) = SNJNT(I3,J3)/SCNT                                  STRS1010
C                                                                       STRS1020
      GAU = SSJNT(I3,3)                                                 STRS1030
      SXPSY = (SSJNT(I3,1)+SSJNT(I3,2))/2.0D0                           STRS1040
      SXMSY = (SSJNT(I3,1)-SSJNT(I3,2))/2.0D0                           STRS1050
      AINT = DSQRT(SXMSY*SXMSY+GAU*GAU)                                 STRS1060
      SSJNT(I3,4) = SXPSY+AINT                                          STRS1070
      SSJNT(I3,5) = SXPSY-AINT                                          STRS1080
      SSJNT(I3,6) = (SSJNT(I3,4)-SSJNT(I3,5))/2.0D0                     STRS1090
      IF (DABS(SXMSY).LE.1.D-10) GO TO 13                               STRS1100
      RAN = DATAN2(GAU,SXMSY)                                           STRS1110
```

```
      GC TO 14
   13 RAN = DSIGN(PI,GAU)/2.0D0
   14 SSJNT(I3,7) = RAN*180.0D0/(2.0D0*PI)
      GAU = SNJNT(I3,3)
      SXPSY = (SNJNT(I3,1)+SNJNT(I3,2))/2.0D0
      SXMSY = (SNJNT(I3,1)-SNJNT(I3,2))/2.0D0
      AINT = DSQRT(SXMSY*SXMSY+GAU*GAU)
      SNJNT(I3,4) = SXPSY+AINT
      SNJNT(I3,5) = SXPSY-AINT
      SNJNT(I3,6) = (SNJNT(I3,4)-SNJNT(I3,5))/2.0D0
   15 SNJNT(I3,7) = SSJNT(I3,7)
C
      WRITE (6,21)
C
      DC 16 I=1,NJT
   16 WRITE (6,20) I,COARD(I,1),COARD(I,2),(SSJNT(I,J),J=1,7)
C
      WRITE (6,22)
C
      DC 17 I=1,NJT
   17 WRITE (6,20) I,COARD(I,1),COARD(I,2),(SNJNT(I,J),J=1,6),SSJNT(I,7)
C
      RETURN
C
   18 FORMAT (5X,I5,6G12.4)
   19 FCRMAT (//,2X,' S-T-R-E-S-S-E-S  /  S-T-R-A-I-N-S  FOR  ELEMENT',I
     13,      //,5X,' JOINT',1X,'SIG X ',6X,'SIG Y ',7X,'TAU ',7X,'EPS
     2X',7X,'EPS Y', 7X,'GAMMA ',//)
   20 FCRMAT (5X,I5,8G12.4,5X,1G12.4)
   21 FORMAT (///,1X,' A-V-E-R-A-G-E  S-T-R-E-S-S-E-S  AT  THE  JOINTS',
     1 //,5X,' JOINT','  X COORD',5X,'Y COORD',5X,'SIG  X ',6X,'SIG Y ',
     27X,'TAU ',7X,'S.MAX',7X,'S.MIN',7X,'T MAX ',12X,'  PHI(DEG.)',//)
   22 FCRMAT (///,1X,' A-V-E-R-A-G-E  S-T-R-A-I-N-S  AT  THE  JOINTS',
     1 //,5X,' JOINT','  X COORD',5X,'Y COORD',5X,'EPS  X ',6X,'EPS Y ',
     27X,'GAMMA',6X,'E.MAX',7X,'E.MIN',7X,'G.MAX ',12X,'  PHI(DEG.)',//)
      END
      SLBROUTINE BCOND (NBAND)
      IMPLICIT REAL*8(A-H,O-Z)
      INPLICIT INTEGER*2(I-N)
      CCMMON /SOL/ BGK(434,86),ALOAD(434),ABGN
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
      CCMMCN /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGL
     1(182),THNP(217)
      PI = 3.14159265359D0
      NEQ = NJT*2
C
      DC 16 I=1,NPBC
```

```
      IJT = NBC(I,1)                                                    BCND 130
      IEQ = IJT*2-1                                                     BCND 140
      KODE = NBC(I,2)                                                   BCND 150
      IDIA = 0                                                          BCND 160
      GO TO (1,3,4,6,10,11,13), KODE                                    BCND 170
    1 IEQ = IEQ+1                                                       BCND 180
    2 BGK(IEQ,1) = ABGN                                                 BCND 190
      IF (IDIA.EQ.1) GO TO 5                                            BCND 200
      GO TO 16                                                          BCND 210
    3 GO TO 2                                                           BCND 220
    4 IDIA = 1                                                          BCND 230
      GO TO 2                                                           BCND 240
    5 IDIA = 0                                                          BCND 250
      GO TO 1                                                           BCND 260
    6 DXY = CLOAD(IJT,2)                                                BCND 270
      IEQ = IEQ+1                                                       BCND 280
    7 ALOAD(IEQ) = ALOAD(IEQ)-BGK(IEQ,1)*DXY                            BCND 290
      BGK(IEQ,1) = ABGN                                                 BCND 300
C                                                                       BCND 310
      DO 9 J=2,NBAND                                                    BCND 320
      IROW = IEQ-J+1                                                    BCND 330
      ICOL = IEQ-1+J                                                    BCND 340
      IF (IROW.LT.1) GO TO 8                                            BCND 350
      ALOAD(IROW) = ALOAD(IROW)-BGK(IROW,J)*DXY                         BCND 360
    8 IF (ICOL.GT.NEQ) GO TO 9                                          BCND 370
      ALOAD(ICOL) = ALOAD(ICOL)-BGK(IEQ,J)*DXY                          BCND 380
    9 CONTINUE                                                          BCND 390
C                                                                       BCND 400
      IF (IDIA.EQ.1) GO TO 12                                           BCND 410
      GO TO 16                                                          BCND 420
   10 DXY = CLOAD(IJT,1)                                                BCND 430
      GO TO 7                                                           BCND 440
   11 IDIA = 1                                                          BCND 450
      DXY = CLOAD(IJT,1)                                                BCND 460
      GO TO 7                                                           BCND 470
   12 IDIA = 0                                                          BCND 480
      GO TO 6                                                           BCND 490
   13 ALPHA = CLOAD(IJT,2)*PI/180.0D0                                   BCND 500
      CAL = DCOS(ALPHA)                                                 BCND 510
      SAL = DSIN(ALPHA)                                                 BCND 520
      IEQP = IEQ+1                                                      BCND 530
C                                                                       BCND 540
      DO 15 J=3,NBAND                                                   BCND 550
      IROW = IEQ-J+2                                                    BCND 560
      JM = J-1                                                          BCND 570
      IF (IROW.LT.1) GO TO 14                                           BCND 580
      A1 = BGK(IROW,JM)*CAL+BGK(IROW,J)*SAL                             BCND 590
      A2 = BGK(IROW,J)*CAL-BGK(IROW,JM)*SAL                             BCND 600
```

```
      BGK(IROW,JM) = A1                                                 BCND 610
      BGK(IROW,J) = A2                                                  BCND 620
   14 IF ((IEQ+J-1).GT.NEQ) GO TO 15                                    BCND 630
      A3 = BGK(IEQ,J)*CAL+BGK(IEQP,JM)*SAL                              BCND 640
      A4 = BGK(IEQP,JM)*CAL-BGK(IEQ,J)*SAL                              BCND 650
      BGK(IEQ,J) = A3                                                   BCND 660
      BGK(IEQP,JM) = A4                                                 BCND 670
   15 CONTINUE                                                          BCND 680
C                                                                       BCND 690
      A1 = BGK(IEQ,1)*CAL*CAL+BGK(IEQ,2)*CAL*SAL*2.0+BGK(IEQP,1)*SAL*SALBCND 700
      A2 = BGK(IEQ,2)*(CAL*CAL-SAL*SAL)+(BGK(IEQP,1)-BGK(IEQ,1))*SAL*CALBCND 710
      BGK(IEQ,1) = A1                                                   BCND 720
      BGK(IEQ,2) = A2                                                   BCND 730
      BGK(IEQP,1) = ABGN                                                BCND 740
   16 CONTINUE                                                          BCND 750
C                                                                       BCND 760
      RETURN                                                            BCND 770
      END                                                               BCND 780
      SUBROUTINE DISPL                                                  DISP  10
      IMPLICIT REAL*8(A-H,O-Z)                                          DISP  20
      IMPLICIT INTEGER*2(I-N)                                           DISP  30
      COMMON /SOL/ BGK(434,86),DISP(434),ABGN                           DISP  40
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRDISP  50
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                         DISP  60
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLDISP  70
     1(182),THNP(217)                                                   DISP  80
      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR                           DISP  90
      DIMENSION REACT(218,2)                                            DISP 100
      EQUIVALENCE (BGK(1,1),REACT(1,1))                                 DISP 110
      PI = 3.14159265359D0                                              DISP 120
C                                                                       DISP 130
      DO 11 I=1,NPBC                                                    DISP 140
      IJT = NBC(I,1)                                                    DISP 150
      IEQ = IJT*2-1                                                     DISP 160
      KODE = NBC(I,2)                                                   DISP 170
      IDIA = 0                                                          DISP 180
      DPBC = 0.0D0                                                      DISP 190
      GO TO (1,3,4,6,7,8,10), KODE                                      DISP 200
    1 IEQ = IEQ+1                                                       DISP 210
      JR = 2                                                            DISP 220
    2 REACT(I,JR) = -DISP(IEQ)*ABGN                                     DISP 230
      DISP(IEQ) = DPBC                                                  DISP 240
      IF (IDIA.EQ.1) GO TO 5                                            DISP 250
      IF (IDIA.EQ.2) GO TO 9                                            DISP 260
      GO TO 11                                                          DISP 270
    3 JR = 1                                                            DISP 280
      GO TO 2                                                           DISP 290
    4 IDIA = 1                                                          DISP 300
```

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      JR = 1                                                            DISP 310
      GO TO 2                                                           DISP 320
    5 ICIA = 0                                                          DISP 330
      GC TO 1                                                           DISP 340
    6 DPBC = CLOAD(IJT,2)                                               DISP 350
      GO TO 1                                                           DISP 360
    7 JR = 1                                                            DISP 370
      DPBC = CLOAD(IJT,1)                                               DISP 380
      GC TO 2                                                           DISP 390
    8 ICIA = 2                                                          DISP 400
      JR = 1                                                            DISP 410
      DPBC = CLOAD(IJT,1)                                               DISP 420
      GC TO 2                                                           DISP 430
    9 ICIA = 0                                                          DISP 440
      DPBC = CLOAD(IJT,2)                                               DISP 450
      GO TO 1                                                           DISP 460
   10 ALPHA = CLOAD(IJT,2)*PI/180.0D0                                   DISP 470
      CAL = DCOS(ALPHA)                                                 DISP 480
      SAL = DSIN(ALPHA)                                                 DISP 490
      A1 = DISP(IEQ)*CAL                                                DISP 500
      A2 = DISP(IEQ)*SAL                                                DISP 510
      IEQP = IEQ+1                                                      DISP 520
      A3 = -DISP(IEQP)*SAL*ABGN                                         DISP 530
      A4 = DISP(IEQP)*CAL*ABGN                                          DISP 540
      DISP(IEQ) = A1                                                    DISP 550
      DISP(IEQP) = A2                                                   DISP 560
      REACT(I,1) = -A3                                                  DISP 570
      REACT(I,2) = -A4                                                  DISP 580
   11 CONTINUE                                                          DISP 590
C                                                                       DISP 600
      MJP = MJT+1                                                       DISP 610
      REACT(MJP,1) = 0.0D0                                              DISP 620
      REACT(MJP,2) = 0.0D0                                              DISP 630
C                                                                       DISP 640
      DC 12 I=1,NPBC                                                    DISP 650
      REACT(MJP,1) = REACT(MJP,1)+REACT(I,1)                            DISP 660
   12 REACT(MJP,2) = REACT(MJP,2)+REACT(I,2)                            DISP 670
C                                                                       DISP 680
      RETURN                                                            DISP 690
      END                                                               DISP 700
```

APPENDIX B

```
C     PROGRAM ISLOAD                                                    MAIN  10
C      *****************************************************************MAIN  20
C      THIS SERVICE PROGRAM FORMS CONSISTENT LOAD VECTORS FOR ANY OF THE MAIN  30
C      FINITE ELEMENTS OF THE PROGRAM  ISANIS                            MAIN  40
C      *****************************************************************MAIN  50
C                                                                       MAIN  60
C      THREE DIFFERENT LOAD VECTORS CAN BE FORMED.                      MAIN  70
C      (A) THERMAL LOAD CORRESPONDING TO A TEMPERATURE DISTRIBUTION     MAIN  80
C      (B) BODY FORCE LOAD/UNIT VOLUME DISTRIBUTED THROUGH AN ELEMENT   MAIN  90
C      (C) INITIAL STRETCH OR CONTRACTION OF AN ELEMENT                 MAIN 100
C                                                                       MAIN 110
C      THE LOAD INTENSITY FUNCTION OVER AN ELEMENT IS DETERMINED BY     MAIN 120
C      ITS NODAL VALUES AND THE SAME SHAPE FUNCTIONS USED IN THE        MAIN 130
C      CONSTRUCTION OF THE ELEMENT ITSELF.  THUS A 4 NODED ELEMENT      MAIN 140
C      WILL ALLOW LINEAR VARIATION, AN 8 NODED ELEMENT QUADRATIC VARIATION MAIN 150
C      AND A 12 NODED ELEMENT CUBIC VARIATION                           MAIN 160
C                                                                       MAIN 170
C      THIS PROGRAM IS AN EXPANSION OF THE PLISOP LOAD GENERATOR        MAIN 180
C      CODED BY GILLES CANTIN IN 1972                                   MAIN 190
C      JAMES M. GILL, MARCH 1975                                        MAIN 200
C     INPUT CARCS NEEDED                                                MAIN 210
C                                                                       MAIN 220
C     TITLE                         (20A4)          ONE CARD PER PROBLEM  MAIN 230
C                                                                       MAIN 240
C     PROBLEM PARAMETERS           (12I5)          ONE CARD PER PROBLEM  MAIN 250
C                                                                       MAIN 260
C      COL.   VARIABLE                                                  MAIN 270
C       1-5      NEL   NUMBER OF ELEMENTS (MAX. IS 182)                 MAIN 280
C       6-10     NJT   NUMBER OF JOINTS (MAX. IS 217)                   MAIN 290
C      11-15     NMAT  NUMBER OF MATERIALS (MAX. IS 10)                 MAIN 300
C      16-20     NCLOAD  NUMBER OF CONCENTRATED LOADS                   MAIN 310
C      21-25     NPBC  NUMBER OF JOINTS WITH BOUNDARY CONDITIONS        MAIN 320
C      26-30     NSTRES  ISOTROPIC MATERIAL PROBLEM TYPE.NSTRES=0 MEANS  MAIN 330
C                          PLANE STRESS, NSTRES = 1 MEANS PLANE STRAIN  MAIN 340
C      31-35     NGP   NUMBER OF GAUSS POINTS DESIRED IN THE INTEGRATION MAIN 350
C      36-40     NTLD  1   IF A THERMAL LOAD IS PRESENT  0  OTHERWISE   MAIN 360
C      41-45     NTBY  1   IF A BODY FORCE LOAD IS PRESENT  0 OTHERWISE MAIN 370
C      46-50     NTIN  1   IF AN INITIAL STRETCH IS TC BE CONSIDERED    MAIN 380
C      51-55     NCARD  MUST BE  1  IF A PUNCED LOAD  VECTOR SUITABLE FOR MAIN 390
C                USE WITH ISANIS IS DESIRED                             MAIN 400
C      56-60     NLEL  NUMBER OF LAMINATE TYPES WHOSE MATERIAL          MAIN 410
C                      PROPERTIES ARE TO BE CALCULATED FROM             MAIN 420
C                      INPUT LAMINA PROPERTIES.  (MAX. 50)              MAIN 430
C      JOINT CARDS      (I5,5X,4G15.8,I5)              NJT CARDS PER PROBLEM MAIN 440
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C    COLUMN                  VARIABLE                                   MAIN 450
C       1-5                  JOINT NUMBER                               MAIN 460
C      11-25                 X COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 470
C                            RADIUS VECTOR LENGTH (POLAR COORD.)        MAIN 480
C      26-40                 Y-COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 490
C                            POLAR ANGLE IN DEGREES (POLAR COORD.)      MAIN 500
C      41-55                 X-COMPONENT OF THE LOAD AT THE JOINT (CART)MAIN 510
C      56-70                 Y-COMPONENT OF THE LOAD AT THE JOINT (CART)MAIN 520
C      70-75                 COORDINATE SYSTEM INDICATOR. IND. = ZERO IFMAIN 530
C                            THE COORDINATE SYSTEM IS CARTESIAN, IF IND MAIN 540
C                            IS NOT ZERO, THE COORDINATE SYSTEM IS POLARMAIN 550
C                                                                       MAIN 560
C    MATERIAL PROPERTIES CARDS(2I5,7F10.2)      NMAT CARDS PER PROBLEM  MAIN 570
C     COLUMN            VARIABLE                                        MAIN 580
C       1-5             MATERIAL NUMBER                                 MAIN 590
C      6-10             INDK=0 FOR ISOTROPIC, INDK=1 FOR ORTHOTROPIC    MAIN 600
C      11-20            YOUNG'S MODULUS (ISOTROPIC MATERIAL)            MAIN 610
C                       E(L) (ORTHOTROPIC MATERIAL)                     MAIN 620
C      21-30            POISSON'S RATIO (ISOTROPIC MATERIAL)            MAIN 630
C                       E(T) (ORTHOTROPIC MATERIAL)                     MAIN 640
C      31-40            NOT USED (ISOTROPIC MATERIAL)                   MAIN 650
C                       NU(LT) (ORTHOTROPIC MATERIAL)                   MAIN 660
C      41-50            NOT USED (ISOTROPIC MATERIAL)                   MAIN 670
C                       NU(TL) (ORTHOTROPIC MATERIAL)                   MAIN 680
C              FOR ORTHOTROPIC MATERIALS, ONLY 3 OF THE ABOVE 4 ENGIN.  MAIN 690
C              CONSTANTS (E(L),E(T),NU(LT),NU(TL)) NEED BE KNOWN. FOR ANMAIN 700
C              UNKNOWN CONSTANT ENTER -1.0 IN THE APPROPRIATE COLUMN.   MAIN 710
C      51-60            NOT USED (ISOTROPIC MATERIALS)                  MAIN 720
C                       G(LT) (ORTHOTROPIC MATERIALS)                   MAIN 730
C      61-70            ALPHA   COEFF. OF ISOTROPIC THERMAL EXPANSION   MAIN 740
C                       NOT USED FOR THERMALLY ANISOTROPIC MATERIALS    MAIN 750
C      71-80            MATERIAL THICKNESS. THIS IS USED FOR ELEMENTS   MAIN 760
C                       WHOSE PROPERTIES ARE NOT TO BE CALCULATED FROM  MAIN 770
C                       INPUT LAMINA PROPERTIES.                        MAIN 780
C                                                                       MAIN 790
C    ELEMENT CARDS           (15I4,F10.5,I5)    NEL CARDS PER PROBLEM   MAIN 800
C       1-4              ELEMENT IDENTIFICATION NUMBER                  MAIN 810
C       5-52             ELEMENT CONNECTIVITY (COUNTER-CLOCKWISE)       MAIN 820
C      53-56            ELEMENT KIND - USE                              MAIN 830
C                        1  FOR A  FOUR  NODAL POINT ELEMENT            MAIN 840
C                        2  FOR AN EIGHT NODAL POINT ELEMENT            MAIN 850
C                        3  FOR A TWELVE NODAL POINT ELEMENT            MAIN 860
C      57-60            MATERIAL/LAMINATE IDENTIFICATION NUMBER         MAIN 870
C      61-70            THETA (DEGREES) USED FOR ORTHOTROPIC MATERIALS  MAIN 880
C                      THETA IS THE ANGLE FROM THE PROBLEM BASED  X AXISMAIN 890
C                       TO  THE MATERIAL BASED L AXIS                   MAIN 900
C      71-75            ENTER 1 FOR A LAMINATED ELEMENT WHOSE           MAIN 910
C                       PROPERTIES ARE TO BE CALCULATED FROM            MAIN 920
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C                       INPUT LAMINA PROPERTIES.                        MAIN 930
C                                                                       MAIN 940
C  BCUNDARY CONDITION CARDS  (2I5)          NPBC  CARDS PER PROBLEM     MAIN 950
C   COL.   VARIABLE                                                     MAIN 960
C    1-5  JOINT NUMBER                                                  MAIN 970
C    6-10 CODE  1 FOR A ROLLER ON AN X AXIS, DISP. IN Y DIRECT. IS ZERO MAIN 980
C               2 FOR A ROLLER ON AN Y AXIS, DISP. IN X DIRECT. IS ZERO MAIN 990
C               3 FOR A FIXED JOINT, BOTH DISP. CCMPONENTS ARE ZERO     MAIN1000
C               4 FOR A ROLLER ON AN X AXIS, THE DISP. IN THE Y DIRECT. MAIN1010
C                  IS SPECIFIED ON THE JOINT CARD IN COL. 41-50         MAIN1020
C               5 FOR A ROLLER ON A Y AXIS, THE DISP. IN THE X DIRECT.  MAIN1030
C                  IS SPECIFIED ON THE JOINT CARD IN COL. 31-40         MAIN1040
C               6 FIXED JOINT WITH DISP. SPECIFIED IN BOTH X AND Y      MAIN1050
C                 DIRECT. ON THE JOINT CARD IN COL.31-40 AND COL.41-50  MAIN1060
C               7 FOR A ROLLER ON AN AXIS INCLINEC ALPHA DEGREES        MAIN1070
C                 WITH RESPECT TO THE X AXIS. THE JOINT CARD MAY        MAIN1080
C                 CONTAIN A TANGENTIAL LOAD IN COL. 31-40 AND THE       MAIN1090
C                 ANGLE ALPHA DEGREES IN COL. 41-50                     MAIN1100
C                                                                       MAIN1110
C   LAMINATE CARD DECK       AT LEAST TWO CARDS PER LAMINATE TYPE       MAIN1120
C                       USED TO CALCULATE THE PROPERTIES OF             MAIN1130
C                       A LAMINATE FROM THE PROPERTIES OF               MAIN1140
C                       THE INDIVIDUAL LAMINAE.                         MAIN1150
C   CARD ONE          2(I5)                                             MAIN1160
C     COLUMN        VARIABLE                                            MAIN1170
C       1-5         LAMTP       LAMINATE TYPE NUMBER (MAX.50)           MAIN1180
C       6-10        LYRNO       NO. LAMINAE IN THIS LAMTP (MAX.100)     MAIN1190
C   CARD TWO       4(I3,F8.3,F9.4)                                      MAIN1200
C     CCLUMN        VARIABLE                                            MAIN1210
C       1-3         MATERIAL NUMBER                                     MAIN1220
C       4-11        LAMINA THICKNESS                                    MAIN1230
C      12-20        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1240
C      21-23        MATERIAL NUMBER                                     MAIN1250
C      24-31        LAMINA THICKNESS                                    MAIN1260
C      32-40        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1270
C      41-43        MATERIAL NUMBER                                     MAIN1280
C      44-51        LAMINA THICKNESS                                    MAIN1290
C      52-60        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1300
C      61-63        MATERIAL NUMBER                                     MAIN1310
C      64-71        LAMINA THICKNESS                                    MAIN1320
C      72-80        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1330
C   AS MANY OF THESE SECOND TYPE CARDS AS NEEDED TO SPECIFY LYRNO LAYERSMAIN1340
C   MAY BE USED                                                         MAIN1350
C   REPEAT THE SEQUENCE WITH NEW CARD TYPE 1 AND 2 AS NEEDED TO         MAIN1360
C   SPECIFY ALL  NLEL LAMINATE TYPES                                    MAIN1370
C  CCNSISTENT LOAD VECTOR INFORMATION   (I5,5F15.0)  NJT CARDS          MAIN1380
C   COL.   VARIABLE                                                     MAIN1390
C    1-5        JOINT NUMBER                                            MAIN1400
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C     6-20      TEMPERATURE AT THE NODE                                 MAIN1410
C    21-35      X COMP. OF BODY FORCE INTENSITY AT THE NODE             MAIN1420
C    36-50      Y COMP. OF BODY FORCE INTENSITY AT THE NODE             MAIN1430
C    51-65      X COMP. OF INITIAL DISP. AT THE NODE                    MAIN1440
C    66-80      Y COMP. OF INITIAL DISP. AT THE NODE                    MAIN1450
C                                                                       MAIN1460
C   AS MANY PROBLEMS AS ONE DESIRES MAY BE STACKED, BUT THE LAST CARD OF MAIN1470
C   A RUN MUST CONTAIN THE WORD    STOP     IN THE FIRST FOUR COLUMNS   MAIN1480
C                                                                       MAIN1490
      IMPLICIT REAL*8(A-H,O-Z)                                          MAIN1500
      IMPLICIT INTEGER*2(I-N)                                           MAIN1510
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRMAIN1520
     1ES,NGP,LM(12),LJT(12),NLEL                                        MAIN1530
      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR                           MAIN1540
      COMMON /FLPL/ COARD(217,2),TLOAD(217,2),ELCON(10,7),TITLE(10),THNPMAIN1550
     1(217),BYNP(217,2),DPIN(217,2),ANGL(182)                           MAIN1560
      COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBYMAIN1570
     1,NTIN,NCARD                                                       MAIN1580
      COMMON /CC/ COORD(12,2),ELAST(3,3)                                MAIN1590
      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                   MAIN1600
      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                        MAIN1610
      DIMENSION FVEC(24), CLOAD(217,2)                                  MAIN1620
      PI = 3.14159265359D0                                              MAIN1630
      MEL = 182                                                         MAIN1640
      MJT = 217                                                         MAIN1650
      MMT = 10                                                          MAIN1660
      MBD = 86                                                          MAIN1670
      MLEL = 50                                                         MAIN1680
      MLYR = 100                                                        MAIN1690
      NZRO = 0                                                          MAIN1700
      ZRO = 0.0D0                                                       MAIN1710
    1 CALL INPUT                                                        MAIN1720
C                                                                       MAIN1730
      DO 2 I=1,MJT                                                      MAIN1740
C                                                                       MAIN1750
      DO 2 J=1,2                                                        MAIN1760
    2 CLOAD(I,J) = ZRO                                                  MAIN1770
C                                                                       MAIN1780
      NN = NSTRES                                                       MAIN1790
      NEQ = 2*NJT                                                       MAIN1800
C                                                                       MAIN1810
      DO 15 I=1,NEL                                                     MAIN1820
      NPEL = NCON(I,15)*4                                               MAIN1830
      NJPE = NPEL*2                                                     MAIN1840
      INDIC = NCON(I,15)                                                MAIN1850
      IF (INDIC.NE.1) GO TO 3                                           MAIN1860
      CALL LAMC (I)                                                     MAIN1870
      GO TO 5                                                           MAIN1880
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```
    3 IMAT = NCON(I,14)                                                 MAIN1890
      THETA = ANGL(I)                                                   MAIN1900
      INDIC = INDK(IMAT)                                                MAIN1910
      CALL STFMAT (IMAT,THETA,INDIC,NSTRES)                             MAIN1920
      IF (INDIC.NE.0) GO TO 4                                           MAIN1930
      ALPHA = ELCON(IMAT,6)                                             MAIN1940
      IF (NN.NE.0) ALPHA = ALPHA*(1.0D0+ELCON(IMAT,2))                  MAIN1950
    4 CCNTINUE                                                          MAIN1960
      TK = ELCON(IMAT,7)                                                MAIN1970
      IF (NSTRES.EQ.1) TK=1.0D0                                         MAIN1980
    5 CCNTINUE                                                          MAIN1990
C                                                                       MAIN2000
      DO 6 J=1,NPEL                                                     MAIN2010
      IJ = NCCN(I,J)                                                    MAIN2020
      LM(J) = IJ                                                        MAIN2030
      TEMP(J) = THNP(IJ)                                                MAIN2040
C                                                                       MAIN2050
      DO 6 JJ=1,2                                                       MAIN2060
      BCFY(J,JJ) = BYNP(IJ,JJ)                                          MAIN2070
      DSPN(J,JJ) = DPIN(IJ,JJ)                                          MAIN2080
    6 COORD(J,JJ) = COARD(IJ,JJ)                                        MAIN2090
C                                                                       MAIN2100
      IF (NTLD.EQ.0) GO TO 9                                            MAIN2110
      LCODE = 1                                                         MAIN2120
      CALL FLOAD (FVEC,NJPE,LCODE,NGP)                                  MAIN2130
      WRITE (6,23) I,NTLD,NZRO,NZRO                                     MAIN2140
C                                                                       MAIN2150
      DO 7 L=1,NPEL                                                     MAIN2160
      IO = 2*L-1                                                        MAIN2170
      IE = IO+1                                                         MAIN2180
      J = NCCN(I,L)                                                     MAIN2190
    7 WRITE (6,24) J,FVEC(IO),FVEC(IE)                                  MAIN2200
C                                                                       MAIN2210
C                                                                       MAIN2220
      DO 8 J=1,NPEL                                                     MAIN2230
      IJ = LM(J)                                                        MAIN2240
      IO = 2*J-1                                                        MAIN2250
      IE = IO+1                                                         MAIN2260
      CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)                                MAIN2270
    8 CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)                                MAIN2280
C                                                                       MAIN2290
    9 IF (NTBY.EQ.0) GO TO 12                                           MAIN2300
      LCODE = 2                                                         MAIN2310
      CALL FLOAD (FVEC,NJPE,LCODE,NGP)                                  MAIN2320
      WRITE (6,23) I,NZRO,NTBY,NZRO                                     MAIN2330
C                                                                       MAIN2340
      DO 10 L=1,NPEL                                                    MAIN2350
      IO = 2*L-1                                                        MAIN2360
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```
      IE = IO+1                                                         MAIN2370
      J = NCON(I,L)                                                     MAIN2380
   10 WRITE (6,24) J,FVEC(IO),FVEC(IE)                                  MAIN2390
C                                                                       MAIN2400
C                                                                       MAIN2410
      DO 11 J=1,NPEL                                                    MAIN2420
      IJ = LM(J)                                                        MAIN2430
      IO = 2*J-1                                                        MAIN2440
      IE = IO+1                                                         MAIN2450
      CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)                                MAIN2460
   11 CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)                                MAIN2470
C                                                                       MAIN2480
   12 IF (NTIN.EQ.0) GO TO 15                                           MAIN2490
      LCODE = 3                                                         MAIN2500
      CALL FLOAD (FVEC,NJPE,LCODE,NGP)                                  MAIN2510
      WRITE (6,23) I,NZRO,NZRO,NTIN                                     MAIN2520
C                                                                       MAIN2530
      DO 13 L=1,NPEL                                                    MAIN2540
      IC = 2*L-1                                                        MAIN2550
      IE = IO+1                                                         MAIN2560
      J = NCON(I,L)                                                     MAIN2570
   13 WRITE (6,24) J,FVEC(IO),FVEC(IE)                                  MAIN2580
C                                                                       MAIN2590
C                                                                       MAIN2600
      DC 14 J=1,NPEL                                                    MAIN2610
      IJ = LM(J)                                                        MAIN2620
      IO = 2*J-1                                                        MAIN2630
      IE = IO+1                                                         MAIN2640
      CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)                                MAIN2650
   14 CLOAD( IJ,2) = FVEC(IE)+CLOAD(IJ,2)                               MAIN2660
C                                                                       MAIN2670
   15 CCNTINUE                                                          MAIN2680
C                                                                       MAIN2690
      KJT = NJT                                                         MAIN2700
C                                                                       MAIN2710
      DC 19 I=1,NPBC                                                    MAIN2720
      NCODE = NBC(I,2)                                                  MAIN2730
      NJT = NBC(I,1)                                                    MAIN2740
      IF ((NCODE.EQ.1).OR.(NCODE.EQ.4)) CLOAD(NJT,2)=ZRO                MAIN2750
      IF ((NCODE.EQ.2).OR.(NCODE.EQ.5)) CLOAD(NJT,1)=ZRO                MAIN2760
      IF ((NCODE.EQ.3).OR.(NCODE.EQ.6)) GO TO 16                        MAIN2770
      GO TO 17                                                          MAIN2780
   16 CLOAD(NJT,1) = ZRO                                                MAIN2790
      CLCAD(NJT,2) = ZRO                                                MAIN2800
   17 IF (NCODE.EQ.7) GO TO 18                                          MAIN2810
      GO TO 19                                                          MAIN2820
   18 ALF = TLOAD(NJT,2)*PI/180.0D0                                     MAIN2830
      COSA = DCOS(ALF)                                                  MAIN2840
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```
      SINA = DSIN(ALF)                                                  MAIN2850
      CLCAD(NJT,1) = CLOAD(NJT,1)*COSA+CLOAD(NJT,2)*SINA                MAIN2860
      CLOAD(NJT,2) = ZRO                                                MAIN2870
   19 CCNTINUE                                                          MAIN2880
C                                                                       MAIN2890
      NJT = KJT                                                         MAIN2900
      WRITE (6,22)                                                      MAIN2910
C                                                                       MAIN2920
      DC 20 I=1,NJT                                                     MAIN2930
      IF (NCARD.NE.0) WRITE (7,21) I,CLOAD(I,1),CLOAD(I,2),THNP(I)      MAIN2940
      WRITE (6,21) I,CLOAD(I,1),CLOAD(I,2),THNP(I)                      MAIN2950
   20 CCNTINUE                                                          MAIN2960
C                                                                       MAIN2970
      GC TO 1                                                           MAIN2980
C                                                                       MAIN2990
   21 FORMAT (5X,1I5,2G25.16,1G15.8)                                    MAIN3000
   22 FORMAT (//,10X,' RESULTANT  LOAD  VECTOR   (NTLD+NTBY+NTIN)',/,   MAIN3010
     1  4X,' JOINT      X  COMPONENT',12X,' Y  COMPONENT',10X,'NODAL TEMMAIN3020
     2P.'//)                                                            MAIN3030
   23 FORMAT (//,5X,'ELEMENT',1I3,'    LOAD VECTOR',3I3,/,5X,'JOINT',7X,'MAIN3040
     1FX',13X,'FY',//)                                                  MAIN3050
   24 FCRMAT (5X,1I5,2G15.6)                                            MAIN3060
      END                                                               MAIN3070
      SUBROUTINE INPUT                                                  INPT  10
      IMPLICIT REAL*8(A-H,O-Z)                                          INPT  20
      IMPLICIT INTEGER*2(I-N)                                           INPT  30
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRINPT  40
     1ES,NGP,LM(12),LJT(12),NLEL                                        INPT  50
      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR                           INPT  60
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),THNPINPT  70
     1(217),BYNP(217,2),DPIN(217,2),ANGL(182)                           INPT  80
      CCMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBYINPT  90
     1,NTIN,NCARD                                                       INPT 100
      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                   INPT 110
      CCMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                        INPT 120
      DATA CHK/'STOP    '/                                              INPT 130
      PI = 3.14159265359D0                                              INPT 140
      ZRC = 0.0D0                                                       INPT 150
      READ (5,25) TITLE                                                 INPT 160
      WRITE (6,26) TITLE                                                INPT 170
      IF (TITLE(1).EQ.CHK) STOP                                         INPT 180
C                                                                       INPT 190
      DC 1 I=1,MJT                                                      INPT 200
      THNP(I) = ZRO                                                     INPT 210
C                                                                       INPT 220
      DO 1 J=1,2                                                        INPT 230
      COARD(I,J) = ZRO                                                  INPT 240
      CLCAD(I,J) = ZRO                                                  INPT 250
```

```
    1 NBC(I,J) = 0
C
C
      DC 2 I=1,MMT
      INDK(I) = 0
C
      DC 2 J=1,7
    2 ELCON(I,J) = ZRO
C
C
      DO 3 I=1,MEL
      ANGL(I) = ZRO
C
      CC 3 J=1,15
    3 NCON(I,J) = 0
C
C
      DO 4 I=1,MLEL
      LYRNO(I) = 0
C
      DO 4 J=1,MLYR
      LAMAT(I,J) = 0
      CRLAM(I,J) = ZRO
    4 TKLAM(I,J) = ZRO
C
C
C     READ PROBLEM PARAMETERS
C
      READ (5,27) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN,NCA
     1RD,NLEL
      WRITE (6,28) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN,NC
     1ARD,NLEL
C
C     READ COORDINATES OF JCINTS AND CONCENTRATED LOADS
C
      WRITE (6,29)
C
      DC 5 I=1,NJT
      READ (5,30) IJT,COARD(IJT,1),COARD(IJT,2),CLOAD(IJT,1),CLOAD(IJT,2
     1),IND
      IF (IND.EQ.0) GO TO 5
      ALF = COARD(IJT,2)*PI/180.0D0
      CCSA = DCOS(ALF)
      SINA = DSIN(ALF)
      XC = COARD(IJT,1)*COSA
      YC = COARD(IJT,1)*SINA
      COARD(IJT,1) = XC
      CCARD(IJT,2) = YC
```

```
    5 CONTINUE                                                          INPT 740
C                                                                       INPT 750
C                                                                       INPT 760
      DO 6 I=1,NJT                                                      INPT 770
    6 WRITE (6,31) I,COARD(I,1),COARD(I,2),CLOAD(I,1),CLOAD(I,2)        INPT 780
C                                                                       INPT 790
C                                                                       INPT 800
C     READ MATERIAL PROPERTIES CARDS                                    INPT 810
C                                                                       INPT 820
C                                                                       INPT 830
      DO 7 I=1,NMAT                                                     INPT 840
    7 READ (5,32) IMAT,INDK(IMAT),(ELCON(IMAT,J),J=1,7)                 INPT 850
C                                                                       INPT 860
      WRITE (6,33)                                                      INPT 870
C                                                                       INPT 880
      DC 8 I=1,NMAT                                                     INPT 890
      WRITE (6,34) I,(ELCON(I,J),J=1,7),INDK(I)                         INPT 900
    8 CONTINUE                                                          INPT 910
C                                                                       INPT 920
C                                                                       INPT 930
C     READ THE ELEMENT CARDS                                            INPT 940
C                                                                       INPT 950
      WRITE (6,35)                                                      INPT 960
      WRITE (6,36)                                                      INPT 970
C                                                                       INPT 980
      DO 9 I=1,NEL                                                      INPT 990
    9 READ (5,37) IJT,(NCON(IJT,J),J=1,14),ANGL(IJT),NCON(IJT,15)       INPT1000
C                                                                       INPT1010
C                                                                       INPT1020
      DO 10 I=1,NEL                                                     INPT1030
   10 WRITE (6,38) I,(NCON(I,J),J=1,14),ANGL(I),NCON(I,15)              INPT1040
C                                                                       INPT1050
C                                                                       INPT1060
C     READ BOUNDARY CONDITION CARDS                                     INPT1070
C                                                                       INPT1080
      WRITE (6,39)                                                      INPT1090
C                                                                       INPT1100
      DO 11 I=1,NPBC                                                    INPT1110
      READ (5,40) (NBC(I,J),J=1,2)                                      INPT1120
   11 WRITE (6,41) (NBC(I,J),J=1,2)                                     INPT1130
C                                                                       INPT1140
C                                                                       INPT1150
      DO 12 I=1,NPBC                                                    INPT1160
C                                                                       INPT1170
      DO 12 J=I,NPBC                                                    INPT1180
      IF (NBC(I,1).LE.NBC(J,1)) GO TO 12                                INPT1190
      NT1 = NBC(I,1)                                                    INPT1200
      NT2 = NBC(I,2)                                                    INPT1210
```

```
      NBC(I,1) = NBC(J,1)
      NBC(I,2) = NBC(J,2)
      NBC(J,1) = NT1
      NBC(J,2) = NT2
   12 CCNTINUE
C
      NSV = 0
C
      DO 13 I=1,NPBC
      IF (NBC(I,2).NE.7) GO TO 13
      NSV = NSV+1
   13 CCNTINUE
C
      NPBCM = NPBC-1
      IF (NSV.EQ.0) GO TO 18
C
      DC 16 I=1,NSV
C
      DO 15 J=1,NPBCM
      IF (NBC(J,2).NE.7) GO TO 15
      NT1 = NBC(J,1)
C
      DO 14 K=J,NPBCM
      KP = K+1
      NBC(K,1) = NBC(KP,1)
   14 NBC(K,2) = NBC(KP,2)
C
      NBC(NPBC,1) = NT1
      NBC(NPBC,2) = 7
      GO TO 16
   15 CONTINUE
C
   16 CONTINUE
C
      WRITE (6,39)
C
      DC 17 I=1,NPBC
   17 WRITE (6,41) (NBC(I,J),J=1,2)
C
   18 CCNTINUE
      IF (NLEL.EQ.0) GO TO 21
C
C     READ LAMINATE CARD DECK
C
C
      DO 19 I=1,NLEL
      READ (5,42) LAMTP,LYRNO(LAMTP)
      L = LYRNO(LAMTP)
```

```
      READ (5,43) ((LAMAT(LAMTP,J),TKLAM(LAMTP,J),ORLAM(LAMTP,J)),J=1,L)INPT1700
   19 CONTINUE                                                          INPT1710
C                                                                       INPT1720
      WRITE (6,44)                                                      INPT1730
C                                                                       INPT1740
      DO 20 I=1,NLEL                                                    INPT1750
      WRITE (6,45) I,LYRNO(I)                                           INPT1760
      LL = LYRNO(I)                                                     INPT1770
C                                                                       INPT1780
      DO 20 J=1,LL                                                      INPT1790
   20 WRITE (6,46) J,LAMAT(I,J),TKLAM(I,J),ORLAM(I,J)                   INPT1800
C                                                                       INPT1810
   21 NLDC = NTLD+NTBY+NTIN                                             INPT1820
      IF (NLDC.EQ.0) GO TO 24                                           INPT1830
C                                                                       INPT1840
C     READ CONSISTENT LOAD VECTOR INFORMATION                           INPT1850
C                                                                       INPT1860
      WRITE (6,47)                                                      INPT1870
C                                                                       INPT1880
      DO 22 I=1,NJT                                                     INPT1890
   22 READ (5,48) II,THNP(II),BYNP(II,1),BYNP(II,2),DPIN(II,1),DPIN(II,2INPT1900
     1)                                                                 INPT1910
C                                                                       INPT1920
C                                                                       INPT1930
      DO 23 I=1,NJT                                                     INPT1940
   23 WRITE (6,49) I,THNP(I),BYNP(I,1),BYNP(I,2),DPIN(I,1),DPIN(I,2)    INPT1950
C                                                                       INPT1960
      RETURN                                                            INPT1970
   24 WRITE (6,50)                                                      INPT1980
      STOP                                                              INPT1990
C                                                                       INPT2000
   25 FORMAT (10A8)                                                     INPT2010
   26 FORMAT (1X,10A8)                                                  INPT2020
   27 FCRMAT (12I5)                                                     INPT2030
   28 FORMAT (//,' NUMBER OF ELEMENTS=',I5,//,' TCTAL NUMBER OF JOINTS='INPT2040
     1,I5,//,' NUMBER OF MATERIALS=',I5,//,' NUMBER OF CONCENTRATED LOADINPT2050
     2S=',I5,//,' NUMBER OF JOINTS WITH BOUNDARY CONDITIONS=',I5,//,' THINPT2060
     3E VALUE OF NSTRES=',I5,//,' NUMBER OF GAUSS POINTS USED ',I5,     INPT2070
     4//,' THERMAL LOAD     ',I5,//,' BODY FORCE LCAD    ',I5,//,       INPT2080
     5' INITIAL STRESS LOAD     ',I5,//,' NCARD',I5,//,'NUMBER OF LAMINATINPT2090
     6E TYPES USED',I5)                                                 INPT2100
   29 FCRMAT (///,' JOINT NUMBER',5X,'X COORDINATE',5X,'Y COORDINATE',7XINPT2110
     1,'X LOAD',9X,' Y LOAD'//)                                         INPT2120
   30 FORMAT (I5,5X,4G15.8,I5)                                          INPT2130
   31 FCRMAT (5X,I3,10X,G14.5,3X,G14.5,5X,G14.5,3X,G14.5)               INPT2140
   32 FCRMAT (2I5,7F10.2)                                               INPT2150
   33 FORMAT (///,' MATERIAL PROPERTIES',//,1X,'MATERIAL    ',2X,'E,E(L)INPT2160
     1',7X,'NU,E(LT)',5X,'NU(LT)',8X,'NU(TL)',10X,'G(LT)',12X,'ALPHA',9XINPT2170
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   2,'THICKNESS',2X,'INDK')                                             INPT2180
 34 FORMAT (' ',1X,I3,5X,7(2X,G13.6),2X,I5)                             INPT2190
 35 FORMAT (///,' ELEMENT CARDS - CONNECTIVITY MATRIX')                 INPT2200
 36 FORMAT (//,' ELEMENT NUMBER',20X,'JOINT NUMBERS',30X,'KIND',3X,'MAINPT2210
   1TERIAL',3X,'THETA(DEG.)',3X,'LAM. IND.')                            INPT2220
 37 FCRMAT (15I4,F10.5,I5)                                              INPT2230
 38 FORMAT (6X,I3,6X,12I5,1X,I5,5X,I5,1X,F10.5,5X,I5)                   INPT2240
 39 FORMAT (///,' BOUNDARY CONDITION CODE',//,5X,'JNT NO',6X,'CODE'//)INPT2250
 40 FCRMAT (2I5)                                                        INPT2260
 41 FCRMAT (5X,I5,I10)                                                  INPT2270
 42 FORMAT (2I5)                                                        INPT2280
 43 FORMAT (4(I3,F8.3,F9.4))                                            INPT2290
 44 FORMAT (///,'LAMINATE PROPERTIES',//,1X,'LAMINATE TYPE',5X,'NO OF INPT2300
   1LAYERS    ',1X,'LAYER NUMBER',5X,'MATERIAL NO.',5X,'THICKNESS',5X,'INPT2310
   2 ORIENTATION'//)                                                    INPT2320
 45 FCRMAT (4X,I5,11X,I5)                                               INPT2330
 46 FCRMAT (30X,I5,8X,I7,10X,F9.3,5X,F9.3)                              INPT2340
 47 FCRMAT (//,' DISTRIBUTED LOAD INFORMATION ',//,                     INPT2350
   1          24X,' BODY FORCE          INITIAL  DISP.',/,2X,           INPT2360
   2 ' JOINT      TEMP.    X COMP.    Y COMP.    X COMP.    Y COMP.',//) INPT2370
 48 FCRMAT (1I5,5F15.0)                                                 INPT2380
 49 FORMAT (4X,1I3,3X,5G10.3)                                           INPT2390
 50 FORMAT (///,5X,' *** NO DISTRIBUTED LOAD *** ')                     INPT2400
    END                                                                 INPT2410
    SLBROUTINE LAMC (IK)                                                LAMC  10
C   THIS SUBROUTINE CALCULATES THE STIFFNESS MATRIX FOR LAMINATED       LAMC  20
C   COMPOSITE MATERIALS. THIS SUBROUTINE ASSUMES THAT THE LAMINATED     LAMC  30
C   COMPOSITE IS OF BALANCED DESIGN SO THAT THE STIFFNESS AND BENDING   LAMC  40
C   FROPERTIES ARE UNCOUPLED.IT CALCULATES AN AVERAGE STIFFNESS MATRIX  LAMC  50
C   FROM THE STIFFNESS PROPERTIES OF EACH LAMINA.                       LAMC  60
C   IK IS THE ELEMENT NUMBER                                            LAMC  70
    IMPLICIT REAL*8(A-H,O-Z)                                            LAMC  80
    IMPLICIT INTEGER*2(I-N)                                             LAMC  90
    CCMMON /FLPL/ COARD(217,2),TLOAD(217,2),ELCON(10,7),TITLE(10),THNPLAMC 100
   1(217),BYNP(217,2),DPIN(217,2),ANGL(182)                             LAMC 110
    CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRLAMC 120
   1ES,NGP,LM(12),LJT(12),NLEL                                          LAMC 130
    COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBYLAMC 140
   1,NTIN,NCARD                                                         LAMC 150
    CCMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                     LAMC 160
    CCMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                          LAMC 170
    CCMMON /CC/ COORD(12,2),ELAST(3,3)                                  LAMC 180
    DIMENSION C(3,3)                                                    LAMC 190
    LAMTP = NCON(IK,14)                                                 LAMC 200
    THETA = ANGL(IK)                                                    LAMC 210
    TK = 0.D0                                                           LAMC 220
C                                                                       LAMC 230
    DC 1 I=1,3                                                          LAMC 240
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C                                                                       LAMC 250
      DO 1 J=1,3                                                        LAMC 260
    1 C(I,J) = 0.D0                                                     LAMC 270
C                                                                       LAMC 280
      LYRS = LYRNO(LAMTP)                                               LAMC 290
C                                                                       LAMC 300
      DO 3 I=1,LYRS                                                     LAMC 310
      MATN = LAMAT(LAMTP,I)                                             LAMC 320
      PHI = THETA+ORLAM(LAMTP,I)                                        LAMC 330
      INDIC = INDK(MATN)                                                LAMC 340
      NSTRES = NN                                                       LAMC 350
      CALL STFMAT (MATN,PHI,INDIC,NSTRES)                               LAMC 360
      TK = TK+TKLAM(LAMTP,I)                                            LAMC 370
C                                                                       LAMC 380
      DO 2 J=1,3                                                        LAMC 390
C                                                                       LAMC 400
      DO 2 K=1,3                                                        LAMC 410
    2 C(J,K) = C(J,K)+ELAST(J,K)*TKLAM(LAMTP,I)                         LAMC 420
C                                                                       LAMC 430
    3 CONTINUE                                                          LAMC 440
C                                                                       LAMC 450
C                                                                       LAMC 460
      DO 4 J=1,3                                                        LAMC 470
C                                                                       LAMC 480
      DO 4 K=1,3                                                        LAMC 490
    4 ELAST(J,K) = C(J,K)/TK                                            LAMC 500
C                                                                       LAMC 510
      RETURN                                                            LAMC 520
      END                                                               LAMC 530
      SUBROUTINE STFMAT (MATN,THETA,INDIC,NSTRES)                       STFM  10
C  THIS SUBROUTINE ASSEMBLES THE ELASTIC STIFFNESS MATRIX FOR A HOOKEAN STFM  20
C  ELEMENT.THE SUBROUTINE CALCULATES AND STORES IN C(3X3)THE VALUES     STFM  30
C  OF THE ELEMENTS OF THIS MATRIX FOR ISOTROPIC MATERIALS UNDER         STFM  40
C  EITHER PLANE STRAIN OR PLANE STRESS OR FOR ANISOTROPIC MATERIALS     STFM  50
C  EXHIBITING TWO DIMENSIONAL ORTHOTROPY.FOR ANISOTROPIC MATERIALS,     STFM  60
C  THE ELEMENTS OF THIS MATRIX ARE FIRST COMPUTED IN THE MATERIAL       STFM  70
C  BASED L-T COORDINATE SYSTEM AND THEN TRANSFORMED TO THE PROBLEM      STFM  80
C  BASED X-Y COORDINATE SYSTEM.                                         STFM  90
      IMPLICIT REAL*8(A-H,O-Z)                                          STFM 100
      IMPLICIT INTEGER*2(I-N)                                           STFM 110
      COMMON /FLPL/ COARD(217,2),TLOAD(217,2),ELCON(10,7),TITLE(10),THNPSTFM 120
     1(217),BYNP(217,2),DPIN(217,2),ANGL(182)                           STFM 130
      COMMON /CC/ COORD(12,2),C(3,3)                                    STFM 140
      DIMENSION A(3,3)                                                  STFM 150
C                                                                       STFM 160
      DO 1 I=1,3                                                        STFM 170
C                                                                       STFM 180
      DO 1 J=1,3                                                        STFM 190
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      A(I,J) = 0.0D0                                                    STFM 200
      C(I,J) = 0.0D0                                                    STFM 210
    1 CONTINUE                                                          STFM 220
C                                                                       STFM 230
      IF (INDIC.EQ.1) GO TO 3                                           STFM 240
      IF (NSTRES.EQ.1) GO TO 2                                          STFM 250
C    FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRESS       STFM 260
C                                                                       STFM 270
      E = ELCCN(MATN,1)                                                 STFM 280
      XNU = ELCON(MATN,2)                                               STFM 290
      E0 = E/(1.0D0-XNU*XNU)                                            STFM 300
      C(1,1) = E0                                                       STFM 310
      C(1,2) = E0*XNU                                                   STFM 320
      C(2,1) = C(1,2)                                                   STFM 330
      C(2,2) = C(1,1)                                                   STFM 340
      C(3,3) = E0*(1.0D0-XNU)/2.0D0                                     STFM 350
      RETURN                                                            STFM 360
    2 CCNTINUE                                                          STFM 370
C                                                                       STFM 380
C   FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRAIN        STFM 390
C                                                                       STFM 400
      E = ELCCN(MATN,1)                                                 STFM 410
      XNU = ELCON(MATN,2)                                               STFM 420
      E1 = E/((1.0D0+XNU)*(1.0D0-2.0D0*XNU))                            STFM 430
      C(1,1) = E1*(1.0D0-XNU)                                           STFM 440
      C(1,2) = E1*XNU                                                   STFM 450
      C(2,1) = C(1,2)                                                   STFM 460
      C(2,2) = C(1,1)                                                   STFM 470
      C(3,3) = E1*(0.5D0-XNU)                                           STFM 480
      RETURN                                                            STFM 490
    3 CCNTINUE                                                          STFM 500
C                                                                       STFM 510
C   FORM THE ANISOTROPIC ORTHOTROPIC ELASTIC MATRIX IN THE MATERIAL BASEDSTFM 520
C   L-T COORDINATE SYSTEM                                               STFM 530
C                                                                       STFM 540
      EL = ELCON(MATN,1)                                                STFM 550
      ET = ELCON(MATN,2)                                                STFM 560
      XNULT = ELCON(MATN,3)                                             STFM 570
      XNUTL = ELCON(MATN,4)                                             STFM 580
      GLT = ELCON(MATN,5)                                               STFM 590
      IF (EL.LT.-0.0D0) EL=XNULT*ET/XNUTL                               STFM 600
      IF (ET.LT.-0.0D0) ET=EL*XNUTL/XNULT                               STFM 610
      IF (XNULT.LT.-0.0D0) XNULT=EL*XNUTL/ET                            STFM 620
      IF (XNUTL.LT.-0.0D0) XNUTL=XNULT*ET/EL                            STFM 630
      Z = 1.0D0-XNULT*XNUTL                                             STFM 640
      A(1,1) = EL/Z                                                     STFM 650
      A(1,2) = ET*XNULT/Z                                               STFM 660
      A(2,1) = A(1,2)                                                   STFM 670
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      A(2,2) = ET/Z                                                     STFM 680
      A(3,3) = GLT                                                      STFM 690
C                                                                       STFM 700
C   THIS ARRAY IS NOW TRANSFORMED FROM THE MATERIAL L-T COORDINATE      STFM 710
C   SYSTEM TO THE PROBLEM X-Y COORDINATE SYSTEM.THE TRANSFORMATION IS   STFM 720
C   ACCCMPLISHED BY ROTATION ABOUT THE Z AXIS THRCUGH THETA DEGREES.    STFM 730
C                                                                       STFM 740
      PI = 3.14159265359D0                                              STFM 750
      THRAD = THETA*PI/180.0D0                                          STFM 760
      CCS1 = DCOS(THRAD)                                                STFM 770
      CCS2 = DCOS(THRAD)**2                                             STFM 780
      COS3 = DCOS(THRAD)**3                                             STFM 790
      COS4 = DCOS(THRAD)**4                                             STFM 800
      SIN1 = DSIN(THRAD)                                                STFM 810
      SIN2 = DSIN(THRAD)**2                                             STFM 820
      SIN3 = DSIN(THRAD)**3                                             STFM 830
      SIN4 = DSIN(THRAD)**4                                             STFM 840
      TWO = 2.0D0                                                       STFM 850
      FOUR = 4.0D0                                                      STFM 860
      C(1,1) = A(1,1)*COS4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*SIN4STFM 870
      C(1,2) = (A(1,1)+A(2,2)-FOUR*A(3,3))*SIN2*COS2+A(1,2)*(SIN4+COS4) STFM 880
      C(1,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN1*COS3+(A(1,2)-A(2,2)+TWO*(STFM 890
     1A(3,3)))*SIN3*COS1                                                STFM 900
      C(2,2) = A(1,1)*SIN4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*COS4STFM 910
      C(2,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN3*COS1+(A(1,2)-A(2,2)+TWO*ASTFM 920
     1(3,3))*SIN1*COS3                                                  STFM 930
      C(3,3) = (A(1,1)+A(2,2)-TWO*(A(1,2)+A(3,3)))*SIN2*COS2+A(3,3)*(SINSTFM 940
     14+COS4)                                                           STFM 950
      C(2,1) = C(1,2)                                                   STFM 960
      C(3,1) = C(1,3)                                                   STFM 970
      C(3,2) = C(2,3)                                                   STFM 980
      RETURN                                                            STFM 990
      ENC                                                               STFM1000
      SLBROUTINE FORMB (B,X,Y,N,DTJ,IND)                                FRMB  10
C                                                                       FRMB  20
C THIS SLBROUTINE FORMS THE   B   MATRIX AS A FUNCTION OF               FRMB  30
C XI AND ETA FOR THE THREE DIFFERENT QUADRILATERAL ELEM. IN THIS FAMILY FRMB  40
C                                                                       FRMB  50
      IMPLICIT REAL*8(A-H,O-Z)                                          FRMB  60
      IMPLICIT INTEGER*2(I-N)                                           FRMB  70
      CIMENSION AJ(2,2), AJIN(2,2), DNX(2,12), W1(2,12), B(3,24)        FRMB  80
      CCMMON /CC/ COORD(12,2),ELAST(3,3)                                FRMB  90
      ZERO = 0.0D0                                                      FRMB 100
C                                                                       FRMB 110
      DO 1 I=1,3                                                        FRMB 120
C                                                                       FRMB 130
      DO 1 J=1,N                                                        FRMB 140
    1 B(I,J) = ZERO                                                     FRMB 150
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C
      ONE = 1.0D0
      TWC = 2.0D0
      FCUR = 4.0D0
      NPT = N/2
C
C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA
C
      IGO = N/8
      GO TO (2,3,4), IGO
C
C    LINEAR FUNCTIONS
C
    2 W1(1,3) = -(ONE-Y)/FOUR
      W1(1,4) = -W1(1,3)
      W1(1,1) = (ONE+Y)/FOUR
      W1(1,2) = -W1(1,1)
      W1(2,3) = -(ONE-X)/FOUR
      W1(2,4) = -(ONE+X)/FOUR
      W1(2,1) = -W1(2,4)
      W1(2,2) = -W1(2,3)
      GO TO 5
    3 CCNTINUE
C
C   GUACRATIC FUNCTIONS
C
      TXPY =  TWO*X+Y
      TXMY =  TWO*X-Y
      TYPX =  TWO*Y+X
      TYMX =  TWO*Y-X
      OMY = ONE-Y
      OPY = ONE+Y
      OPX = ONE+X
      OMX = ONE-X
      W1(1,5) = OMY*TXPY/FOUR
      W1(1,6) = -OMY*X
      W1(1,7) = OMY*TXMY/FOUR
      W1(1,8) = OPY*OMY/TWO
      W1(1,1) = OPY*TXPY/FOUR
      W1(1,2) = -OPY*X
      W1(1,3) = OPY*TXMY/FOUR
      W1(1,4) = -OPY*OMY/TWO
      W1(2,5) = OMX*TYPX/FOUR
      W1(2,6) = -OPX*OMX/TWO
      W1(2,7) = OPX*TYMX/FOUR
      W1(2,8) = -Y*OPX
      W1(2,1) = OPX*TYPX/FOUR
      W1(2,2) = OPX*OMX/TWO
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      W1(2,3) = OMX*TYMX/FOUR                                           FRMB 640
      W1(2,4) = -Y*OMX                                                  FRMB 650
      GO TO 5                                                           FRMB 660
    4 CONTINUE                                                          FRMB 670
C                                                                       FRMB 680
C    CUBIC FUNCTIONS                                                    FRMB 690
C                                                                       FRMB 700
      OPX = ONE+X                                                       FRMB 710
      OPY = ONE+Y                                                       FRMB 720
      OMX = ONE-X                                                       FRMB 730
      OMY = ONE-Y                                                       FRMB 740
      TPTMNX = 3.0D0+2.0D0*X-9.0D0*X*X                                  FRMB 750
      TMTMNX = 3.0D0-2.0D0*X-9.0D0*X*X                                  FRMB 760
      TPTMNY = 3.0D0+2.0D0*Y-9.0D0*Y*Y                                  FRMB 770
      TMTMNY = 3.0D0-2.0D0*Y-9.0D0*Y*Y                                  FRMB 780
      TYPO = 3.0D0*Y+ONE                                                FRMB 790
      TYMO = 3.0D0*Y-ONE                                                FRMB 800
      TXPO = 3.0D0*X+ONE                                                FRMB 810
      TXMO = 3.0D0*X-ONE                                                FRMB 820
      TMTPEX = 10.0D0-27.0D0*X*X+18.0D0*X-9.0D0*Y*Y                     FRMB 830
      TMTMEX = 10.0D0-27.0D0*X*X-18.0D0*X-9.0D0*Y*Y                     FRMB 840
      TMNPEY = 10.0D0-9.0D0*X*X+18.0D0*Y-27.0D0*Y*Y                     FRMB 850
      TMNMEY = 10.0D0-9.0D0*X*X-18.0D0*Y-27.0D0*Y*Y                     FRMB 860
      TTT = 32.0D0                                                      FRMB 870
      TTN = 32.0D0/9.0D0                                                FRMB 880
      W1(1,7) = OMY*TMTPEX/TTT                                          FRMB 890
      W1(1,8) = -TPTMNX*OMY/TTN                                         FRMB 900
      W1(1,9) = TMTMNX*OMY/TTN                                          FRMB 910
      W1(1,10) = -OMY*TMTMEX/TTT                                        FRMB 920
      W1(1,11) = -OPY*OMY*TYMO/TTN                                      FRMB 930
      W1(1,12) = OPY*OMY*TYPO/TTN                                       FRMB 940
      W1(1,1) = -OPY*TMTMEX/TTT                                         FRMB 950
      W1(1,2) = TMTMNX*OPY/TTN                                          FRMB 960
      W1(1,3) = -TPTMNX*OPY/TTN                                         FRMB 970
      W1(1,4) = OPY*TMTPEX/TTT                                          FRMB 980
      W1(1,5) = -OPY*OMY*TYPO/TTN                                       FRMB 990
      W1(1,6) = OPY*OMY*TYMO/TTN                                        FRMB1000
      W1(2,7) = OMX*TMNPEY/TTT                                          FRMB1010
      W1(2,8) = OPX*OMX*TXMO/TTN                                        FRMB1020
      W1(2,9) = -OPX*OMX*TXPO/TTN                                       FRMB1030
      W1(2,10) = OPX*TMNPEY/TTT                                         FRMB1040
      W1(2,11) = -TPTMNY*OPX/TTN                                        FRMB1050
      W1(2,12) = TMTMNY*OPX/TTN                                         FRMB1060
      W1(2,1) = -OPX*TMNMEY/TTT                                         FRMB1070
      W1(2,2) = OPX*OMX*TXPO/TTN                                        FRMB1080
      W1(2,3) = -OPX*OMX*TXMO/TTN                                       FRMB1090
      W1(2,4) = -OMX*TMNMEY/TTT                                         FRMB1100
      W1(2,5) = OMX*TMTMNY/TTN                                          FRMB1110
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      W1(2,6) = -OMX*TPTMNY/TTN                                         FRMB1120
    5 CCNTINUE                                                          FRMB1130
C                                                                       FRMB1140
C FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)                        FRMB1150
C                                                                       FRMB1160
C                                                                       FRMB1170
      DO 6 I=1,2                                                        FRMB1180
C                                                                       FRMB1190
      DO 6 J=1,2                                                        FRMB1200
      AJ(I,J) = ZERO                                                    FRMB1210
C                                                                       FRMB1220
      DC 6 K=1,NPT                                                      FRMB1230
    6 AJ(I,J) = AJ(I,J)+W1(I,K)*COORD(K,J)                              FRMB1240
C                                                                       FRMB1250
C                                                                       FRMB1260
C CALCULATE DETERMINANT OF THE JACOBIAN                                 FRMB1270
C                                                                       FRMB1280
      DTJ = AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)                             FRMB1290
      IF (IND.NE.0) RETURN                                              FRMB1300
C                                                                       FRMB1310
C INVERT JACOBIAN IN AJIN                                               FRMB1320
C                                                                       FRMB1330
      AJIN(1,1) = AJ(2,2)/DTJ                                           FRMB1340
      AJIN(1,2) = -AJ(1,2)/DTJ                                          FRMB1350
      AJIN(2,1) = -AJ(2,1)/DTJ                                          FRMB1360
      AJIN(2,2) = AJ(1,1)/DTJ                                           FRMB1370
C                                                                       FRMB1380
C FORM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y       FRMB1390
C                                                                       FRMB1400
C                                                                       FRMB1410
      DC 7 I=1,2                                                        FRMB1420
C                                                                       FRMB1430
      DO 7 J=1,NPT                                                      FRMB1440
      DNX(I,J) = ZERO                                                   FRMB1450
C                                                                       FRMB1460
      DO 7 K=1,2                                                        FRMB1470
    7 DNX(I,J) = DNX(I,J)+AJIN(I,K)*W1(K,J)                             FRMB1480
C                                                                       FRMB1490
C                                                                       FRMB1500
C FORM B(I,J) MATRIX                                                    FRMB1510
C                                                                       FRMB1520
C                                                                       FRMB1530
      DC 8 I=1,NPT                                                      FRMB1540
      ICD = 2*I-1                                                       FRMB1550
      IEV = 2*I                                                         FRMB1560
      B(1,IOD) = DNX(1,I)                                               FRMB1570
      B(2,IEV) = DNX(2,I)                                               FRMB1580
      B(3,IEV) = DNX(1,I)                                               FRMB1590
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    8 B(3,IOD) = DNX(2,I)                                               FRMB1600
C                                                                       FRMB1610
      RETURN                                                            FRMB1620
      END                                                               FRMB1630
      SUBROUTINE FLOAD (FVEC,N,LCCDE,NGP)                               FLOD  10
C   FLOAD IS A NUMERICAL INTEGRATION SUBROUTINE THAT FORMS THE          FLOD  20
C   CONSISTENT LOAD VECTORS FOR ANY OF THE THREE ELEMENTS OF THIS       FLOD  30
C   FAMILY OF ISOPARAMETRIC ELEMENTS                                    FLOD  40
C   GAUSSIAN QUADRATURE WITH A MAXIMUM OF FIVE ORDINATES IS USED.       FLOD  50
C   FVEC(24) IS THE OUTPUT ARRAY CONTAINING THE LOAD VECTOR UPON EXIT   FLOD  60
C   N IS EQUAL TO TWICE THE NUMBER OF NODAL POINTS IN THE ELEMENT       FLOD  70
C   LCCDE MUST BE EQUAL TO  1 FOR A TEMPERATURE LOAD,                   FLOD  80
C                           2 FOR A BODY FORCE LOAD VECTOR,             FLOD  90
C                           3 FOR AN INITIAL DEFORMATION LOAD VECTOR.   FLOD 100
C        NGP IS THE NUMBER OF GAUSS POINTS USED FOR INTEGRATION         FLOD 110
C        COORD(12,2) IS AN ARRAY CONTAINING THE COORDINATES OF ALL      FLOD 120
C                    THE NODAL POINTS OF THE ELEMENT TO BE CONSTRUCTED  FLOD 130
C        ELAST(3,3)  IS AN ARRAY CONTAINING ALL THE ELASTIC CONSTANTS   FLOD 140
C                    OF THE ELEMENT, THE ORDER OF THE STRESS COMPONENTS FLOD 150
C                    IS: SIGMAX,SIGMAY,TAUXY                            FLOD 160
C        TEMP(12) IS A VECTOR OF NODAL TEMPERATURES                     FLOD 170
C        BDFY(12,2) IS A VECTOR OF NODAL VALUES OF THE BODY FORCE       FLOD 180
C        DSPN(12,2) IS A VECTOR OF INITIAL VALUES OF DISPLACEMENTS      FLOD 190
C        TK IS THE THICKNESS OF THE ELEMENT ( UNITY FOR PLANE STRAIN)   FLOD 200
C        ALPHA IS THE COEFFICIENT OF EXPANSION OF THE MATERIAL          FLOD 210
C        NN IS EQUAL TO 1 FOR PLANE STRAIN,0 FOR PLANE STRESS           FLOD 220
C        NTLD IS EQUAL TO  1 FOR A THERMAL LOAD,   0 OTHERWISE          FLOD 230
C        NTBY IS EQUAL TO  1 FOR A BODY FORCE LOAD,0 OTHERWISE          FLOD 240
C        NTIN IS EQUAL TO  1 FOR AN IN.STR. LOAD,  0 OTHERWISE          FLOD 250
C        NCARD IS THE PUNCH CONTROL CARD                                FLOD 260
C   CODED BY PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL   FLOD 270
      IMPLICIT REAL*8(A-H,O-Z)                                          FLOD 280
      IMPLICIT INTEGER*2(I-N)                                           FLOD 290
      COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBYFLOD 300
     1,NTIN,NCARD                                                       FLOD 310
      COMMON /CC/ COORD(12,2),ELAST(3,3)                                FLOD 320
      DIMENSION FVEC(24), VAL(12), BTD(24,3), WVEC(24), SK(24,24), B(3,2FLOD 330
     14)                                                                FLOD 340
      DIMENSION DSPO(24)                                                FLOD 350
      DIMENSION XI(5), AI(5), AIA(5,5)                                  FLOD 360
      DIMENSION X2(2), X3(3), X4(4), X5(5), A2(2), A3(3), A4(4), A5(5)  FLOD 370
      DATA X2/.577350269189626D0,-.577350269189626D0/                   FLOD 380
      DATA A2/1.0D0,1.0D0/                                              FLOD 390
      DATA X3/.774596669241483D0,0.0D0,-.774596669241483D0/             FLOD 400
      DATA A3/.555555555555556D0,.888888888888889D0,.555555555555556D0/ FLOD 410
      DATA X4/.8611363115940D0,.3399810435848D0,-.3399810435848D0,-.8611FLOD 420
     1363115940D0/                                                      FLOD 430
      DATA A4/.3478548451374D0,.6521451548625D0,.6521451548625D0,.347854FLOD 440
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     18451374D0/                                                        FLOD 450
      DATA X5/.9061798459386D0,.5384693101056D0,0.0D0,-.5384693101056D0,FLOD 460
     1-.9061798459386D0/                                                FLOD 470
      CATA A5/.2369268850561D0,.4786286704993D0,.5688888888888D0,.478628FLOD 480
     16704993D0,.2369268850561D0/                                       FLOD 490
      IF (NGP.EQ.0) NGP = 5                                             FLOD 500
      IF (TK.EQ.0.0D0) TK=1.0D0                                         FLOD 510
C                                                                       FLOD 520
      DC 1 I=1,24                                                       FLOD 530
      FVEC(I) = 0.0D0                                                   FLOD 540
    1 WVEC(I) = 0.0D0                                                   FLOD 550
C                                                                       FLOD 560
      JUMP = NGP-1                                                      FLOD 570
C                                                                       FLOD 580
      DC 6 I=1,NGP                                                      FLOD 590
      GO TO (2,3,4,5), JUMP                                             FLOD 600
    2 XI(I) = X2(I)                                                     FLOD 610
      AI(I) = A2(I)                                                     FLOD 620
      GO TO 6                                                           FLOD 630
    3 XI(I) = X3(I)                                                     FLOD 640
      AI(I) = A3(I)                                                     FLOD 650
      GC TO 6                                                           FLOD 660
    4 XI(I) = X4(I)                                                     FLOD 670
      AI(I) = A4(I)                                                     FLOD 680
      GO TO 6                                                           FLOD 690
    5 XI(I) = X5(I)                                                     FLOD 700
      AI(I) = A5(I)                                                     FLOD 710
    6 CCNTINUE                                                          FLOD 720
C                                                                       FLOD 730
C                                                                       FLOD 740
      DC 7 I=1,NGP                                                      FLOD 750
C                                                                       FLOD 760
      DO 7 J=1,NGP                                                      FLOD 770
    7 AIA(I,J) = AI(I)*AI(J)                                            FLOD 780
C                                                                       FLOD 790
      NPEL = N/2                                                        FLOD 800
C                                                                       FLOD 810
      DC 24 I=1,NGP                                                     FLOD 820
      X = XI(I)                                                         FLOD 830
C                                                                       FLOD 840
      DO 24 J=1,NGP                                                     FLOD 850
      Y = XI(J)                                                         FLOD 860
      GC TO (8,13,17), LCODE                                            FLOD 870
    8 ILT = 0                                                           FLOD 880
      CALL FORMB (B,X,Y,N,DTJ,ILT)                                      FLOD 890
C                                                                       FLOD 900
      DO 9 II=1,N                                                       FLOD 910
C                                                                       FLOD 920
```

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      DO 9 JJ=1,3                                                       FLOD 930
      BTD(II,JJ) = 0.0D0                                                FLOD 940
C                                                                       FLOD 950
      DO 9 KK=1,3                                                       FLOD 960
    9 BTD(II,JJ) = BTD(II,JJ)+B(KK,II)*ELAST(KK,JJ)                     FLOD 970
C                                                                       FLOD 980
      CALL SHAPE (VAL,X,Y,NPEL)                                         FLOD 990
      CFT = 0.0D0                                                       FLOD1000
C                                                                       FLOD1010
      DC 10 II=1,NPEL                                                   FLCD1020
   10 CFT = CFT+VAL(II)*TEMP(II)                                        FLOD1030
C                                                                       FLOD1040
      CFT = CFT*ALPHA*TK*DTJ                                            FLOD1050
C                                                                       FLOD1060
      DO 11 II=1,N                                                      FLOD1070
   11 WVEC(II) = (BTD(II,1)+BTD(II,2))*CFT                              FLOD1080
C                                                                       FLOD1090
C                                                                       FLOD1100
      DC 12 II=1,N                                                      FLOD1110
   12 FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)                             FLOD1120
C                                                                       FLOD1130
      GC TO 24                                                          FLOD1140
   13 CALL SHAPE (VAL,X,Y,NPEL)                                         FLOD1150
      ILT = 1                                                           FLOD1160
      CALL FORMB (B,X,Y,N,DTJ,ILT)                                      FLOD1170
      CFTX = 0.0D0                                                      FLOD1180
      CFTY = 0.0D0                                                      FLOD1190
C                                                                       FLOD1200
      DC 14 II=1,NPEL                                                   FLOD1210
      CFTX = CFTX+VAL(II)*BDFY(II,1)                                    FLOD1220
   14 CFTY = CFTY+VAL(II)*BDFY(II,2)                                    FLOD1230
C                                                                       FLOD1240
C                                                                       FLOD1250
      DC 15 II=1,NPEL                                                   FLOD1260
      IC = 2*II-1                                                       FLOD1270
      IE = IO+1                                                         FLOD1280
      WVEC(IO) = VAL(II)*CFTX*DTJ*TK                                    FLOD1290
   15 WVEC(IE) = VAL(II)*CFTY*DTJ*TK                                    FLOD1300
C                                                                       FLOD1310
C                                                                       FLOD1320
      DC 16 II=1,N                                                      FLOD1330
   16 FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)                             FLOD1340
C                                                                       FLOD1350
      GC TO 24                                                          FLOD1360
   17 ILT = 0                                                           FLOD1370
      CALL FORMB (B,X,Y,N,DTJ,ILT)                                      FLOD1380
C                                                                       FLOD1390
      DO 18 II=1,N                                                      FLOD1400
```

```
C                                                                       FLOD1410
      DO 18 JJ=1,3                                                      FLOD1420
      BTD(II,JJ) = 0.0D0                                                FLOD1430
C                                                                       FLOD1440
      DO 18 KK=1,3                                                      FLOD1450
   18 BTD(II,JJ) = BTD(II,JJ)+B(KK,II)*ELAST(KK,JJ)                     FLOD1460
C                                                                       FLOD1470
C                                                                       FLOD1480
      DO 19 II=1,N                                                      FLOD1490
C                                                                       FLOD1500
      DO 19 JJ=1,N                                                      FLOD1510
      SK(II,JJ) = 0.0D0                                                 FLOD1520
C                                                                       FLOD1530
      DO 19 KK=1,3                                                      FLOD1540
   19 SK(II,JJ) = SK(II,JJ)+BTD(II,KK)*B(KK,JJ)                         FLOD1550
C                                                                       FLOD1560
C                                                                       FLOD1570
      DC 20 II=1,NPEL                                                   FLOD1580
      IC = 2*II-1                                                       FLOD1590
      IE = IO+1                                                         FLOD1600
      DSPO(IO) = DSPN(II,1)                                             FLOD1610
   20 DSPO(IE) = DSPN(II,2)                                             FLOD1620
C                                                                       FLOD1630
C                                                                       FLOD1640
      DO 21 II=1,N                                                      FLOD1650
      WVEC(II) = 0.0D0                                                  FLOD1660
C                                                                       FLOD1670
      DC 21 JJ=1,N                                                      FLOD1680
   21 WVEC(II) = WVEC(II)+SK(II,JJ)*DSPC(JJ)                            FLOD1690
C                                                                       FLOD1700
C                                                                       FLOD1710
      DC 22 II=1,N                                                      FLOD1720
   22 WVEC(II) = -WVEC(II)*TK*DTJ                                       FLOD1730
C                                                                       FLOD1740
C                                                                       FLOD1750
      DC 23 II=1,N                                                      FLOD1760
   23 FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)                             FLOD1770
C                                                                       FLOD1780
   24 CCNTINUE                                                          FLOD1790
C                                                                       FLOD1800
      RETURN                                                            FLOD1810
      END                                                               FLOD1820
      SUBROUTINE SHAPE (VAL,X,Y,NPEL)                                   SHPE  10
      IMPLICIT REAL*8(A-H,O-Z)                                          SHPE  20
      IMPLICIT INTEGER*2(I-N)                                           SHPE  30
      DIMENSION VAL(12), XYL(4,2), XYQ(8,2), XYC(12,2), IPERM(4)        SHPE  40
      DATA XYL/1.0D0,-1.0D0,-1.0D0,1.0D0,1.0D0,1.0D0,-1.0D0,-1.0D0/     SHPE  50
      DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,SHPE  60
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     11.0D0,1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0/                     SHPE  70
      DATA XYC/1.0D0,.3333333333333D0,-.3333333333333D0,-1.0D0,-1.0D0,-1SHPE  80
     1.0D0,-1.0D0,-.3333333333333D0,.3333333333333D0,1.0D0,1.0D0,1.0D0,1SHPE  90
     2.0D0,1.0D0,1.0D0,1.0D0,.3333333333333D0,-.3333333333333D0,-1.0D0,-SHPE 100
     31.0D0,-1.0D0,-1.0D0,-.3333333333333D0,.3333333333333D0/           SHPE 110
      FL(X,Y,X1,Y1) = (ONE+X*X1)*(ONE+Y*Y1)/FOUR                        SHPE 120
      FQC(X,Y,X1,Y1) = (ONE+X*X1)*(ONE+Y*Y1)*(X*X1+Y*Y1-ONE)/FOUR       SHPE 130
      FQM(X,Y,Y1) = (ONE-X*X)*(ONE+Y*Y1)/TWO                            SHPE 140
      FCC(X,Y,X1,Y1) = (ONE+X*X1)*(ONE+Y*Y1)*(.9D1*(X*X+Y*Y)-.1D2)/3.2D1SHPE 150
      FCM(X,Y,X1,Y1) = (ONE-X*X)*(ONE+.9D1*X*X1)*(ONE+Y*Y1)*.9D1/3.2D1  SHPE 160
      ONE = 1.0D0                                                       SHPE 170
      TWO = 2.0D0                                                       SHPE 180
      FOUR = 4.0D0                                                      SHPE 190
      IGO = NPEL/4                                                      SHPE 200
      GO TO (1,3,7), IGO                                                SHPE 210
    1 CONTINUE                                                          SHPE 220
C        LINEAR FUNCTIONS                                               SHPE 230
C                                                                       SHPE 240
      DO 2 I=1,4                                                        SHPE 250
      X1 = XYL(I,1)                                                     SHPE 260
      Y1 = XYL(I,2)                                                     SHPE 270
    2 VAL(I) = FL(X,Y,X1,Y1)                                            SHPE 280
C                                                                       SHPE 290
      RETURN                                                            SHPE 300
    3 CONTINUE                                                          SHPE 310
C        QUADRATIC FUNCTIONS                                            SHPE 320
C                                                                       SHPE 330
      DO 4 I=1,7,2                                                      SHPE 340
      X1 = XYQ(I,1)                                                     SHPE 350
      Y1 = XYQ(I,2)                                                     SHPE 360
    4 VAL(I) = FQC(X,Y,X1,Y1)                                           SHPE 370
C                                                                       SHPE 380
C                                                                       SHPE 390
      DO 5 I=2,6,4                                                      SHPE 400
      Y1 = XYQ(I,2)                                                     SHPE 410
    5 VAL(I) = FQM(X,Y,Y1)                                              SHPE 420
C                                                                       SHPE 430
C                                                                       SHPE 440
      DO 6 I=4,8,4                                                      SHPE 450
      X1 = XYQ(I,1)                                                     SHPE 460
    6 VAL(I) = FQM(Y,X,X1)                                              SHPE 470
C                                                                       SHPE 480
      RETURN                                                            SHPE 490
    7 CONTINUE                                                          SHPE 500
C        CUBIC FUNCTIONS                                                SHPE 510
C                                                                       SHPE 520
      DO 8 I=1,10,3                                                     SHPE 530
      X1 = XYC(I,1)                                                     SHPE 540
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```
      Y1 = XYC(I,2)                                                     SHPE 550
    8 VAL(I) = FCC(X,Y,X1,Y1)                                           SHPE 560
C                                                                       SHPE 570
      IPERM(1) = 2                                                      SHPE 580
      IPERM(2) = 3                                                      SHPE 590
      IPEKM(3) = 8                                                      SHPE 600
      IPERM(4) = 9                                                      SHPE 610
C                                                                       SHPE 620
      DO 9 I=1,4                                                        SHPE 630
      IJ = IPERM(I)                                                     SHPE 640
      X1 = XYC(IJ,1)                                                    SHPE 650
      Y1 = XYC(IJ,2)                                                    SHPE 660
    9 VAL(IJ) = FCM(X,Y,X1,Y1)                                          SHPE 670
C                                                                       SHPE 680
      IPERM(1) = 5                                                      SHPE 690
      IPERM(2) = 6                                                      SHPE 700
      IPERM(3) = 11                                                     SHPE 710
      IPERM(4) = 12                                                     SHPE 720
C                                                                       SHPE 730
      DO 10 I=1,4                                                       SHPE 740
      IJ = IPERM(I)                                                     SHPE 750
      X1 = XYC(IJ,1)                                                    SHPE 760
      Y1 = XYC(IJ,2)                                                    SHPE 770
   10 VAL(IJ) = FCM(Y,X,Y1,X1)                                          SHPE 780
C                                                                       SHPE 790
      RETURN                                                            SHPE 800
      END                                                               SHPE 810
```

REFERENCES

1. Eisenmann, J.R., and others, "Toward Reliable Composites: An Examination of Design Methodology," Journal of Composite Materials, v. 7, p. 298-307, July 1973.

2. Kaminski, B.E., "NARMCO 5208/T300 - A Graphite-Epoxy System Engineered for High Performance Aircraft Structures," Proceedings of the National SAMPE Technical Conference, v. 5, 1973.

3. Calcote, L.R., The Analysis of Laminated Composite Structures, p. 15-38, Von Nostrand Reinhold Company, 1969.

4. Toth, I.J., Brentnall, W.D., and Menke, G.D., "Fabricating Aluminum Matrix Composites," Journal of Metals, v. 24, No. 12, p. 19-22, September 1972.

5. Hersh, M.S. and Featherby, M., "Joining of Boron/Aluminum Composites," Journal of Aircraft, v. 10, No. 2, p. 99-104, February 1973.

6. Waddoups, M.E., "Characterization of Advanced Composite Materials for Structural Design," Polymer Engineering and Science, v. 15, No. 3, p. 160-165, March 1975.

7. Eisenmann, J.R. and Kaminski, B.E., "Fracture Control for Composite Structures," Engineering Fracture Mechanics, v. 4, p. 907-913, 1972.

8. Waddoups, M.E., Chief Design Engineer, Advanced Composites Group, General Dynamics Corporation, Fort Worth Texas; private communication with the author, July, 1975.

9. Laboratory tests performed at Convair Division, General Dynamics Corporation, results distributed internally, November, 1974.

10. Cozzone, F.P., Melcon, M.A., and Hoblit, F.M., "Analysis of Lugs and Shear Pins made of Aluminum or Steel Alloys," Product Engineering, v. 21, No. 5, p. 113-117, May 1950.

11. Laboratory tests performed at Convair Division, General Dynamics Corporation, results distributed internally, August, 1974.

12. Waddoups, M.E., Eisenmann, J.R., and Kaminski, B.E., "Macroscopic Fracture Mechanics of Advanced Composite

Materials," Journal of Composite Materials, v. 5, p. 446-454, October 1971.

13. Griffith, A.A., "The Phenomena of Rupture and Flow in Solids," Philosophical Transactions of the Royal Society, v. 221A, p. 163-198, 1920.

14. Eisenmann, J.R., "Bolted Joint Static Strength Model for Advanced Composite Materials," paper presented at the Third Conference on Fibrous Composites in Flight Vehicle Design, Williamsburg, Virginia, 4-6 November 1975.

15. Laboratory tests performed at Convair Division, General Dynamics Corporation, results distributed internally, January 1975.

16. Lewith, C.W., Chesson, E.Jr., and Munse, W.H., "Riveted and Bolted Joints: Fatigue of Bolted Structural Connections," Journal of the Structural Division Proceedings of the American Society of Civil Engineers, v. 89, No. ST1, p. 49-65, February 1963.

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